EEE 352

Introduction to Power System 3 credit

Introduction to conventional and renewable energy resources for Power generation

Principles of Power generations hydro and steam Plants, gas turbine plant, Magnetohydrodynamic (MHD) generation, economic considerations in the Choice of plant types.

Power Supply planning.

- System planning: generating Station Location and plant size, high, medium, Low voltage power network, busbar systems, substation sitting, Load, Voltage and Power factor control Load diversity and Utilization factors, Maximum demand. System economics - Economic loading, Choice of Machines, tariffs overhead lines: Long, medium and short line Calculations Power Charts. Transmission line efficiency and Voltage regulations P.U. notation

Power Cables

Transformers: Operating Characteristics, loading, losses, efficiency and regulation, winding types and connection equivalent circuits, three winding transformers, tap changing

Distribution System
Distribution system planning, Choice of distribution voltages, radial characteristics, subtransmission & distribution substations

Electromagnetic wave cause interference
(Name of Author) superscript.

Symmetrical Components

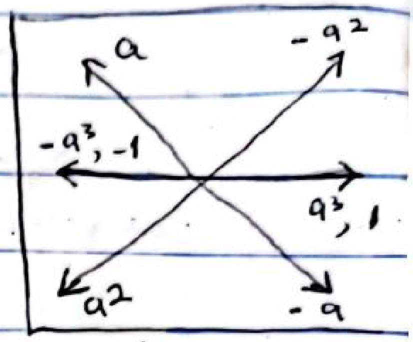
an unbalanced system of n related phasors can be resolved into n systems of balanced phasors called symmetrical components of the original phasors.

Equal in length & angle b/w them is equal

1 Positive - sequence components consisting of three phasors equal in magnitude displaced from each other by 120° in phase & having the same phase sequence as the original phasors

2 Negative sequence components consisting of three phasors equal in magnitude displaced from each other by 120° in phase, and having the phase sequence opposite to that of the original phasor

Zero - sequence components, consisting of three phasors equal in magnitude and with zero phase displacement from each other



abc   acb

+ve   -ve

1 ≡ +ve   2 - ve   0 zero

$$V_a = V_{a1} + V_{a2} + V_{a0} \quad \text{(i)}$$

$$V_b = V_{b1} + V_{b2} + V_{b0} \quad \text{(ii)}$$

$$V_c = V_{c1} + V_{c2} + V_{c0} \quad \text{(iii)}$$

Three sets of balanced phasors which are the symmetrical components of the unbalanced phasors are shown below.

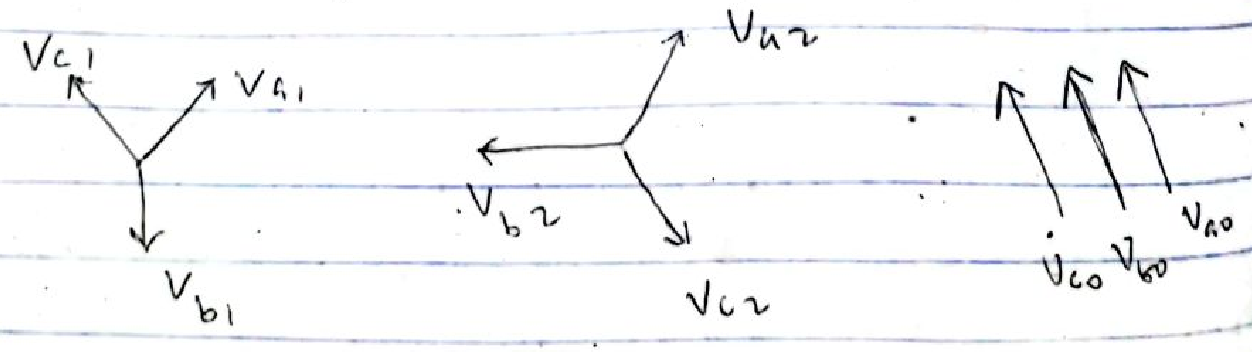


Fig. 1

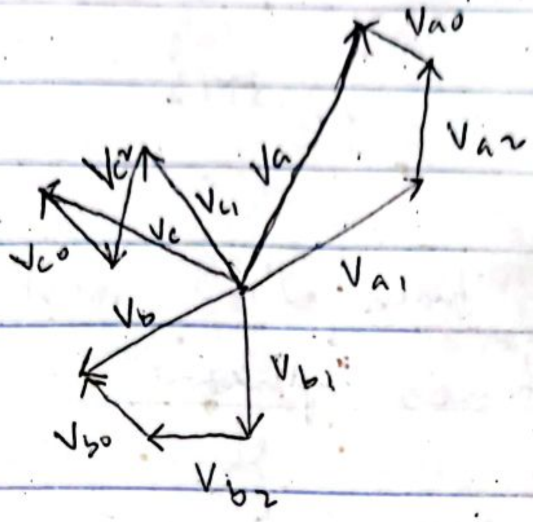


Fig 2

Graphical addition of the components shown in Fig 1 to obtain three unbalanced phasors

Multiplication of two complex numbers
{Product of their magnitude & the sum of their angles}

j – Causes 90° rotation
↑+ve
-1 – causes rotation 180°
two successive multiplication by j Causes rotation of $90^\circ + 90^\circ$ hence $j \times j$ causes 180° rotation $j^2 \equiv -1$

a Causes rotation of 120° in the counter clockwise direction

$a = 1\angle 120 = 1e^{j2\pi/3} = -0.5 + j0.866$

apply 'a' twice 240° rotation
three times phasor is rotated 360°

$a^2 = 1\angle 240° = -0.5 - j0.866$
$a^3 = 1\angle 360° = 1\angle 0° = 1$

$V_{b1} = a^2 V_{a1}$ $\quad V_{c1} = aV_{a1}$
$V_{b2} = a V_{a2}$ $\quad V_{c2} = a^2 V_{a2}$ (a)
$V_{b0} = V_{a0}$ $\quad V_{c0} = V_{a0}$

repeat i
ults a in $V_a = V_{a1} + V_{a2} + V_{a0}$ (b)
ii & iii $V_b = a^2 V_{a1} + aV_{a2} + V_{a0}$ (c)
$V_c = aV_{a1} + a^2 V_{a2} + V_{a0}$ (d)

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \quad \text{(e)}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \quad (f)$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \quad (g)$$

Pre multiplying both sides of eqn for V_a, V_b, V_c by A^{-1}

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (h)$$

$$V_{a0} = \tfrac{1}{3}(V_a + V_b + V_c) \quad (i)$$
$$V_{a1} = \tfrac{1}{3}(V_a + aV_b + a^2 V_c) \quad (j)$$
$$V_{a2} = \tfrac{1}{3}(V_a + a^2 V_b + a V_c) \quad (k)$$

V_{b0}, V_{b1}, V_{b2}, V_{c0}, V_{c1} & V_{c2} can be gotten from (a) ie eqn (a)

No zero sequence exists if the sum of the ~~sm~~ unbalanced phasors is zero from eqn (i)

Line to line voltage in a 3 ϕ system is always zero. Zero sequence voltages are never present in line voltages regardless of imbalance.

The sum of the 3 line to neutral voltages Phasors is not necessary ~~to~~ zero; hence voltages to neutral may contain zero sequence components

analytically or graphically

$$I_a = I_{a1} + I_{a2} + I_{a0}$$
$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0}$$
$$I_c = a I_{a1} + a^2 I_{a2} + I_{a0}$$

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) \quad *$$
$$I_{a1} = \frac{1}{3}(I_a + a I_b + a^2 I_c)$$
$$I_{a2} = \frac{1}{3}(I_a + a^2 I_b + a I_c)$$

CamScanner

In a 3ϕ system, the sum of the line currents is equal to the current I_n in the return path through the neutral

thus $I_a + I_b + I_c = I_n$ $**$

Combining $*$ & $**$ gives

$$I_n = 3 I_{a0}$$

In the absence of a path through the neutral of a 3 phase $I_n = 0$ & the line current contains no zero sequence component.

A; Δ connected load eg. eg.

The effect on Impedence

$$V_{abc} = Z_{abc} I_{abc} \quad (1)$$

where Z_{abc} is a 3×3 matrix giving the self and mutual impedances in and between phases

However

$$V_{abc} = [A]\, V_{012} \quad (2)$$

&

$$I_{abc} = [A]\, I_{012} \quad (3)$$

Subs 2 & 3 in 1

$$[A]\, V_{012} = Z_{abc}\, [A]\, I_{012} \quad (4)$$

or

$$V_{012} = [A]^{-1}\, Z_{abc}\, [A]\, I_{012} \quad (5)$$

Define

$$Z_{012} = [A]^{-1}\, Z_{abc}\, A \quad (6)$$

So that

$$V_{012} = Z_{012}\, I_{012} \quad 7$$

For a typical power system, the matrix Z_{abc} is not diagonal, but has possesses certain symmetries. These symmetries are such that Z_{012} is diagonal either exactly or approximately. When this is the case, the analysis is greatly simplified. An example illustra

Power Considerations

The reversal rule of matrix algebra states that the transpose of the product of two matrices is equal to the product of the transpose of the matrices in reverse order. Hence $[AV]^T = V^T A^T$

Let

$$S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^* \qquad (1)$$

In matrix notation

t ≡ transpose
* ≡ conjugate

$$S_{3\phi} = V_{abc\,t}\, I^*_{abc} \qquad (2)$$

$$= \{[A] V_{012}\}_t \{[A] I_{012}\}^* \qquad (3)$$

$$= V_{012\,t}\, A_t\, A^*\, I^*_{012} \qquad (4)$$

$$A_t\, A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

Therefore

$$S_{3\phi} = 3 V_{012t} I^*_{012} \quad (6)$$

$$S_{3\phi} = 3(V_0 I_0^* + V_1 I_1^* + V_2 I_2^*) \quad (7)$$

Notice there are no cross terms or mixed quantities in the expression for $S_{3\phi}$. This is useful when equivalent circuits are being considered.

Eg.

Evaluate $S_{3\phi}$

given $V_{abc} = \begin{bmatrix} 100 \\ -100 \\ 0 \end{bmatrix}$ and $I_{abc} = \begin{bmatrix} j10 \\ -10 \\ -10 \end{bmatrix}$

use eqns 2 & 7 & compare the results.

$$S_{3\phi} = V_{abct} I^*_{abc}$$

$$= \begin{bmatrix} 100 & -100 & 0 \end{bmatrix} \begin{bmatrix} -j10 \\ -10 \\ -10 \end{bmatrix} = 1000 - j1000$$

Now

$$V_{012} = A^{-1} V_{abc}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 100 \\ -100 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 0 \\ 100\angle -30^\circ \\ 100\angle +30 \end{bmatrix}.$$

$$I_{012} = A^{-1} I_{abc}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} j10 \\ -10 \\ -10 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} j10-20 \\ j10+10 \\ j10+10 \end{bmatrix}$$

$$S_{3\phi} = 3\, V_{012_t}\, I^*_{012}$$

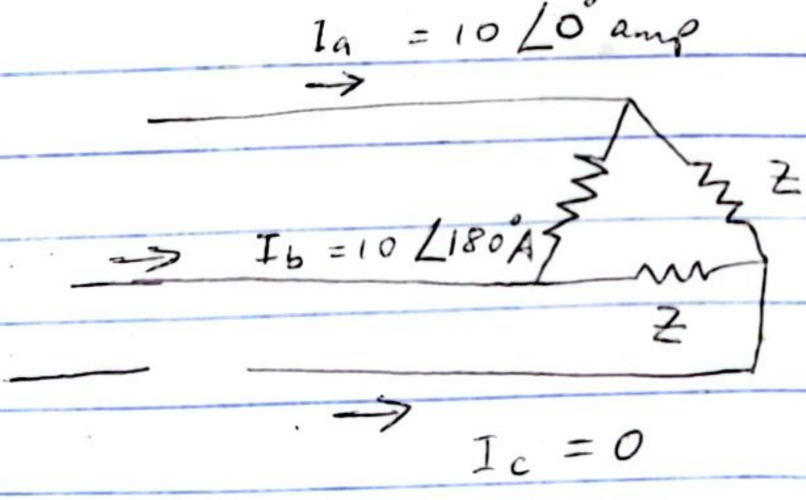
$$= \begin{bmatrix} 0 & \frac{100}{\sqrt{3}}\angle -30^\circ & \frac{100}{\sqrt{3}}\angle +30^\circ \end{bmatrix}\begin{bmatrix} -j10-20 \\ 10\sqrt{2}\angle -45^\circ \\ 10\sqrt{2}\angle -45^\circ \end{bmatrix}$$

$$\frac{1000\sqrt{2}}{\sqrt{3}}\left[1\angle -75^\circ + 1\angle -15^\circ\right]$$

$$= 1.000 - j1000$$

Example

One conductor of a three phase line is open. The current flowing to the Δ-connected load through line a is 10A. With the current in line a as reference and assuming that line c is open, find the symmetrical components of the line currents.



Giving the diagram above

$I_a = 10\angle 0° A$ $\quad I_b$

Assume; since the third line is open hence since c is reference $\angle 0$ & the other $\angle 180$

using eqns for I_{a0}, I_{a1} & I_{a2}, *

$$I_{a0} = \frac{1}{3}(10\angle 0^\circ + 10\angle 180^\circ + 0) = 0$$

$$I_{a1} = \frac{1}{3}(10\angle 0^\circ + 10\angle 180^\circ + 120^\circ + 0)$$

$$= 5 - j2.89 = 5.78\angle -30^\circ A$$

$$I_{a2} = \frac{1}{3}(10\angle 0^\circ + 10\angle 180^\circ + 240^\circ + 0)$$

$$= 5 + j2.89 = 5.78\angle 30^\circ A$$

from eqn (a)

$$I_{b1} = 5.78\angle -150^\circ A$$

$$I_{b2} = 5.78\angle 150 A$$

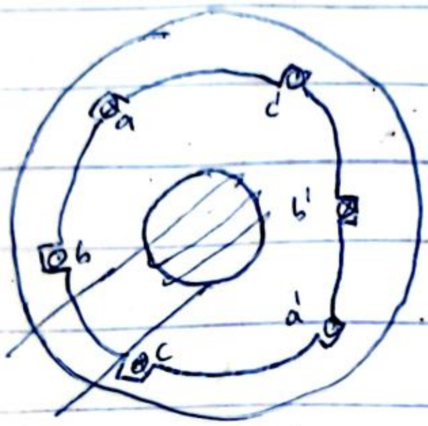
$$I_{b0} = 0$$

$$I_{c1} = 5.78\angle 90^\circ A$$

$$I_{c2} = 5.78\angle -90^\circ A$$

$$I_{c0} = 0$$

Note that components I_{c1} and I_{c2} have definite values although line c is open and can carry no net current. As is expected, therefore, the sum of the components in line c is zero. Of course the sum of the components in line a is $10\angle 0^\circ A$ and the sum of the components in line b is $10\angle 180° A$.



θ_d : angular displacement θ_d measured from the axis of coil a used to define positions round the armature.

Hence mmf around the armature produced by armature current will be a function of both displacement θ_d & time t.

Circuit Model of a Synchronous Machine

consider phase a. We wish to develope the phasor diagram. visualize the separate components of the flux at the axis of coil a. here, the θ_d is zero ∴ The rotor flux ϕ_f is the only one to be considered when the armature current is zero. This flux ϕ_f generates the no-load voltage E_{ao} which we shall call E_f

The flux ϕ_{ar} due to the armature-reaction mmf F_{ar} is in phase with the current i_a (at $\theta_d = 0$)

$$i_a = I_m \cos \omega t$$ I_m - maxi I

The sum of three sinusoidal terms of equal magnitude displaced in phase by 120° and 240° is zero

The armature mmf produced by the three phases a, b, c,

$$F_{ar} = F_a + F_b + F_c = \frac{3}{2} F_m \cos(\theta_d - \omega t)$$

Since at the axis $\theta_d = 0$

This is to show that they are in phase ie $I \cos\theta_d$ $F \cos\theta_d$ $= F \cos -\theta_d$

& $\cos \omega t = \cos(-\omega t)$, the current I_a & F_{ar} are in phase. & thus the flux it produces.

The sum of ϕ_f & ϕ_{ar} is ϕ_r neglecting saturation.

This is the resultant flux which generates the voltage E_r in the coil windings composing phase a.

The phasor diagram shows that voltages E_f and E_{ar} lag by $90°$ the fluxes ϕ_f and ϕ_{ar} which generate them.
The resultant flux ϕ_r is the flux across the air gap of the machine and generates E_r in the stator
The same analysis can be done for each phase.

This is

to show that
they are
in phase

i.e $I \cos \theta_1$

$F \cos \theta_d$

$= F \cos -\theta_d$

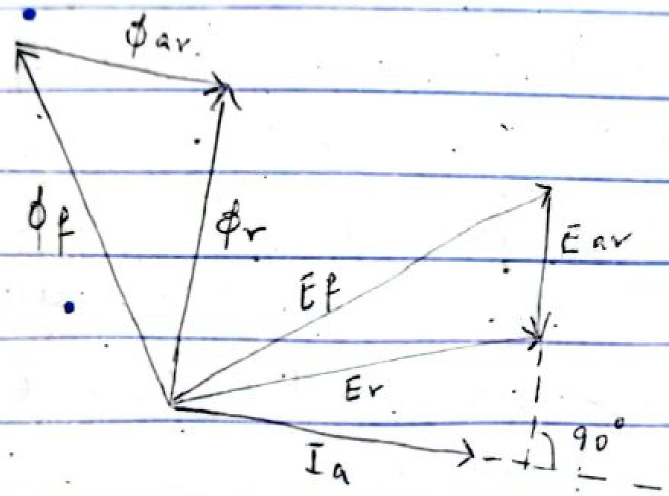
ϕ_f : is the flux produced by the dc current in the rotor

ϕ_{ar} : is the flux produced by armature reaction

A rotating sinusoidal flux has maximum linkage with a coil when the direction of the mmf causing the flux ~~linkage maximum~~ coincides with the axis of the coil, but the rate of change of flux linkage is then zero. Likewise when the rotor has turned through 90° the flux linkage becomes zero but then rate of change is a maximum.

Thus the voltage induced in the coil is 90° out of phase with the flux linkages.

Applying lenz's law and taking into account the assumed positive directions of current in the coil and mmf it could be shown that the ~~induction~~ induced voltage is lagging the mmf



Phasor diagram showing time relation of components of flux at the axis of coil a ($\theta_d = 0$) and the voltages and currents of phase a of the generator. Similar diagrams may be drawn for phase b & c and apply to all cylindrical rotor generators.

E_{ar} is lagging I_a by $90°$ the magnitude of E_{ar} is determined by ϕ_{ar} which is proportional to $|I_a|$. Since it is produced by the armature current. An inductive reactance X_{ar} can be specified such that

$$E_{ar} = -j I_a X_{ar}$$

This equation is defined so that E_{ar} would have the desired phase angle of $90°$ lagging with respect to I_a

Voltage generated in phase a by air gap flux is E_r

$$E_r = E_f + E_{ar} = E_f - jI_a X_{ar}$$

Voltage generated by resultant flux exceeds the terminal voltage V_t of the phase only by the drop due to the armature current times the leakage ~~resist~~ reactance X_L of the winding if resistance is neglected.

Given terminal voltage V_t

$$V_t = E_r - jI_a X_L$$

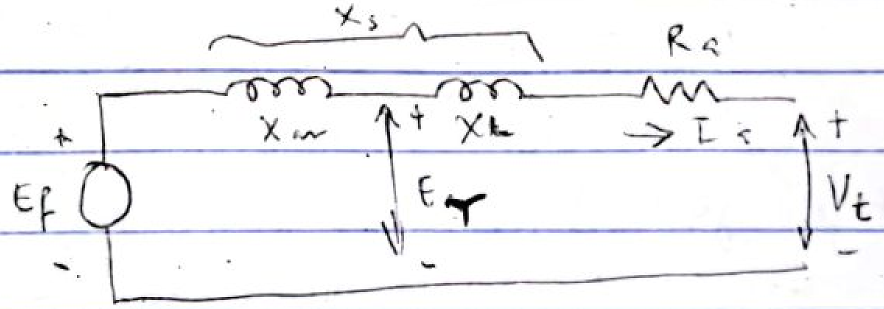
$I_a X_L$ accounts for the voltage drop caused by the portion of the flux (produced by armature current) which does not cross the air gap of the machine

$$V_t = E_f - jI_aX_{ar} - jI_aX_l$$

generated at no load | due to armature reaction | due to armature leakage reactance

$$V_t = E_f - jI_aX_s$$

$$X_s = X_{ar} + X_l \quad \text{(synchronous reactance)}$$



Equivalent circuit of an ac gen

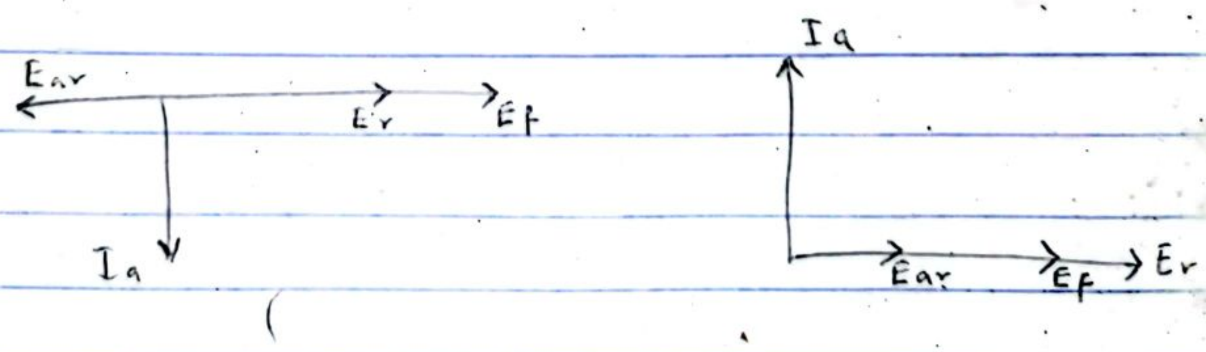
$$V_t = E_f - I_a(R_a + jX_s)$$

R_a = armature resistance

usually very small relative to X_s.

and usually omitted

When the current I_a lags the no-load generated voltage E_f by $90°$, ϕ_{ar} subtracts directly from ϕ_f, and ϕ_r is greatly reduced. Conversely, armature current leading the no-load voltage by $90°$ causes ϕ_{ar} to add directly to ϕ_f, and ϕ_r is greatly increased. See fig below. If a highly inductive load is applied to a generator the terminal voltage will be considerably below the no-load terminal voltage. On the other hand a capacitive load will cause the terminal voltage to rise considerably above its no-load value.



Phasor diagrams showing the relation between E_f and E_{ar} when current delivered by a generator is (a) lagging E_f by $90°$ and (b) leading E_f by $90°$.

Per Unit System

A means of ~~mor~~ normalizing dimensioned electrical quantities.

The basic per unit scaling equation is

$$\text{Per Unit Value} = \frac{\text{Actual Value}}{\text{Base Value}}$$

$$S = VI^*$$

Let

$$V_{base} I_{base} = S_{base}$$

$$\frac{S\angle\phi}{S_{base}} = \frac{V\angle\alpha \; I\angle-\beta}{V_{base} \cdot I_{base}}.$$

$$S_{pu} = V_{pu} I_{pu}^*$$

Let

$$Z_{base} = \frac{V_{base}}{I_{base}}$$

$$= \frac{V_{base}^2}{S_{base}}$$

$$Z_{pu} = \frac{V_{pu}}{I_{pu}}$$

$$V_{LN_{base}} = \frac{V_{Lbase}}{\sqrt{3}}$$

$$V_{Lpu} = \frac{V_L}{V_{Lbase}}$$

$$V_{LNpu} = \frac{V_{LN}}{V_{LNbase}}$$

$$V_{LN} = \frac{V_L}{\sqrt{3}}$$

$$V_{LNpu} = \frac{V_{LN}}{V_{LNbase}}$$

$$= \frac{V_L / \sqrt{3}}{V_{Lbase} / \sqrt{3}}$$

$$= \frac{V_L}{V_{Lbase}}$$

$$= V_{Lpu}$$

In power calculations, 3ϕ power is usually stated.

$$S_{1\phi_{base}} = S_{3\phi_{base}} / 3$$

$$Z_{pu} = \frac{V_{pu}}{I_{pu}}$$

$$V_{LN\,base} = \frac{V_{L\,base}}{\sqrt{3}}$$

$$V_{L\,pu} = \frac{V_L}{V_{L\,base}}$$

$$V_{LN\,pu} = \frac{V_{LN}}{V_{LN\,base}}$$

$$V_{LN} = \frac{V_L}{\sqrt{3}}$$

$$V_{LN\,pu} = \frac{V_{LN}}{V_{LN\,base}}$$

$$= \frac{V_L/\sqrt{3}}{V_{L\,base}/\sqrt{3}}$$

$$= \frac{V_L}{V_{L\,base}}$$

$$= V_{L\,pu}.$$

In power calculations, 3ϕ power is usually stated.

$$S_{1\phi\,base} = S_{3\phi\,base}/3$$

$$S_{3\phi} = 3 S_{1\phi}$$

$$S_{1\phi pu} = \frac{S_{1\phi}}{S_{1\phi base}}$$

$$= \frac{S_{3\phi}/3}{S_{3\phi base}/3}$$

$$= S_{3\phi pu}$$

The wye impedance base is

$$Z_{Ybase} = \frac{V^2_{LN base}}{S_{1\phi base}}$$

$$= \frac{\left[V_{Lbase}/\sqrt{3}\right]^2}{S_{3\phi base}/3}$$

$$= \frac{V^2_{Lbase}}{S_{3\phi base}}$$

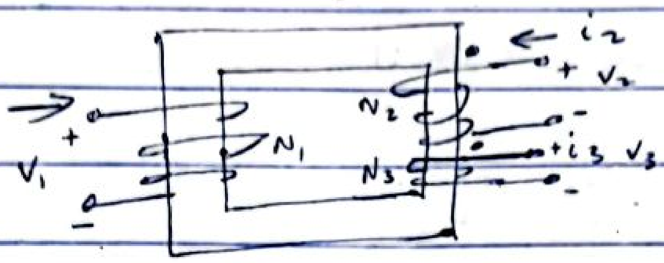
$$Z_{\Delta base} = 3 Z_{Ybase}$$

$$Z_{Ypu} = Z_{\Delta pu}$$

line current base

$$I_{Lbase} = \frac{S_{3\phi base}}{V_{Lbase}\sqrt{3}}$$ because $S_{3\phi} = \sqrt{3} V_L I_L$

Transformer Equivalent Circuit.



assuming infinit conductivity & permeability
hence no resistance & no flux leakage.
As a result of infinit conductivity, no resistance, hence

$$V_1 = N_1 \frac{d\phi}{dt}$$

$$V_2 = N_2 \frac{d\phi}{dt}$$

$$V_3 = N_3 \frac{d\phi}{dt}$$

Since all ϕ are same & there is no leakage due to infinit permeability

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

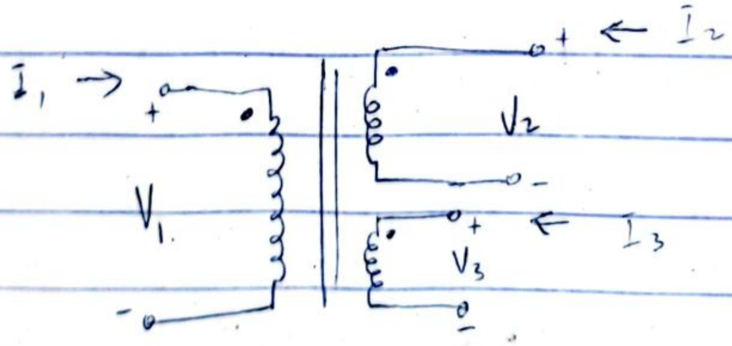
$$\frac{V_2}{V_3} = \frac{N_2}{N_3}$$

$$\frac{V_3}{V_1} = \frac{N_3}{N_1}$$

This means that

$$V_1 = \frac{N_1}{N_2} V_2$$
$$V_2 = \frac{N_2}{N_3} V_3$$
$$V_3 = N_3/N_1 \; V_1$$

Information on how the coils are wound is usually not given, hence, polarity marks (dots) are provided



From

Ampere's Law

$$\oint \hat{H} \cdot \hat{dl} = i_{enclosed}$$

$$\frac{V_3}{V_1} = \frac{N_3}{N_1}$$

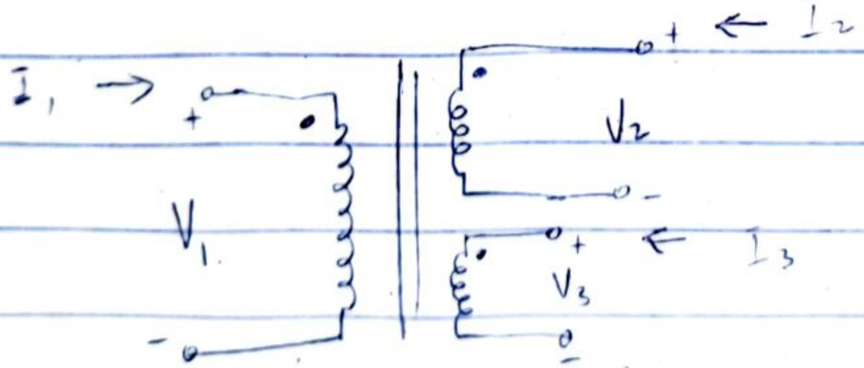
This means that

$$V_1 = \frac{N_1}{N_2} V_2$$

$$V_2 = \frac{N_2}{N_3} V_3$$

$$V_3 = N_3/N_1 \; V_1$$

Information on how the coils are wound is usually not given, hence, polarity marks (dots) are provided



From

Ampere's Law

$$\oint \hat{H} \cdot \hat{dl} = i_{enclosed}$$

If the core center line is chosen as the enclosed path, the magnetic field intensity is zero everywhere because of infinite core permeability.

$$0 = i_{\text{enclosed}}$$

$$\cancel{0 = Ni}$$

$$0 = N_1 i_1 + N_2 i_2 + N_3 i_3$$

(mmf)

$$N_1 \bar{I}_1 + N_2 \bar{I}_2 + N_3 \bar{I}_3 = 0$$

$$S = V_1 \bar{I}_1^* + V_2 \bar{I}_2^* + V_3 \bar{I}_3^*$$

$$= V_1 \bar{I}_1^* + \frac{N_2}{N_1} V_1 \bar{I}_2^* + \frac{N_3}{N_1} V_1 \bar{I}_3^*$$

$$= \frac{V_1}{N_1} \left[N_1 \bar{I}_1 + N_2 \bar{I}_2 + N_3 \bar{I}_3 \right]^*$$

$$= 0$$

Magnetic field intensity
H

$$V_{2base} = \frac{N_2}{N_1} V_{1base}$$

$$V_{3base} = \frac{N_3}{N_1} V_{1base}$$

$$S_{1base} = S_{2base} = S_{3base} = S_{1\phi base}$$

$$I_{1base} = \frac{S_{1\phi base}}{V_{1base}}$$

$$I_{2base} = \frac{S_{1\phi base}}{V_{2base}}$$

$$I_{3base} = \frac{S_{1\phi base}}{V_{3base}}$$

$$I_{2base} = \frac{N_1}{N_2} I_{1base}$$

$$I_{3base} = \frac{N_1}{N_3} I_{1base}$$

$$V_1 = \frac{N_1}{N_2} V_2$$

$$\frac{V_1}{V_{1base}} = \frac{(N_1/N_2)V_2}{V_{1base}}$$

$$V_{1pu} = V_{2pu}$$

$$\frac{V_1}{V_{1base}} = \frac{(N_1/N_3)\,V_3}{(N_1/N_3)\,V_{3base}}$$

$$V_{1pu} = V_{3pu}$$

$$V_{1pu} = V_{2pu} = V_{3pu} \qquad *$$

$$I_1 + (N_2/N_1)I_2 + (N_3/N_1)I_3 = 0$$

$$\frac{I_1}{I_{base}} + \frac{(N_2/N_1)I_2}{I_{1base}} + \frac{(N_3/N_1)I_3}{I_{1base}} = 0$$

$$\frac{I_1}{I_{1base}} + \frac{(N_2/N_1)I_2}{(N_2/N_1)I_{2base}} + \frac{(N_3/N_1)I_3}{(N_3/N_1)I_{3base}} = 0$$

$$I_{1pu} + I_{2pu} + I_{3pu} = 0 \qquad **$$

Eqn * & ** can be used to construct an equivalent circuit.

$$V_{1pu} = V_{2pu}$$

$$\frac{V_1}{V_{1base}} = \frac{(N_1/N_3)\,V_3}{(N_1/N_3)\,V_{3base}}$$

$$V_{1pu} = V_{3pu}$$

$$V_{1pu} = V_{2pu} = V_{3pu} \qquad *$$

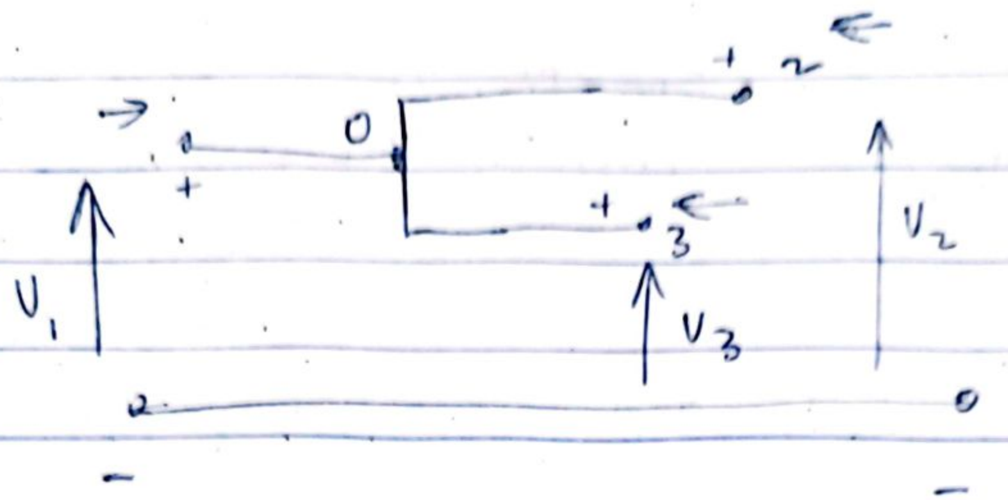
$$I_1 + (N_2/N_1)\,I_2 + (N_3/N_1)\,I_3 = 0$$

$$\frac{I_1}{I_{base}} + \frac{(N_2/N_1)\,I_2}{I_{1base}} + \frac{(N_3/N_1)\,I_3}{I_{1base}} = 0$$

$$\frac{I_1}{I_{1base}} + \frac{(N_2/N_1)\,I_2}{(N_2/N_1)\,I_{2base}} + \frac{(N_3/N_1)\,I_3}{(N_3/N_1)\,I_{3base}} = 0$$

$$I_{1pu} + I_{2pu} + I_{3pu} = 0 \qquad **$$

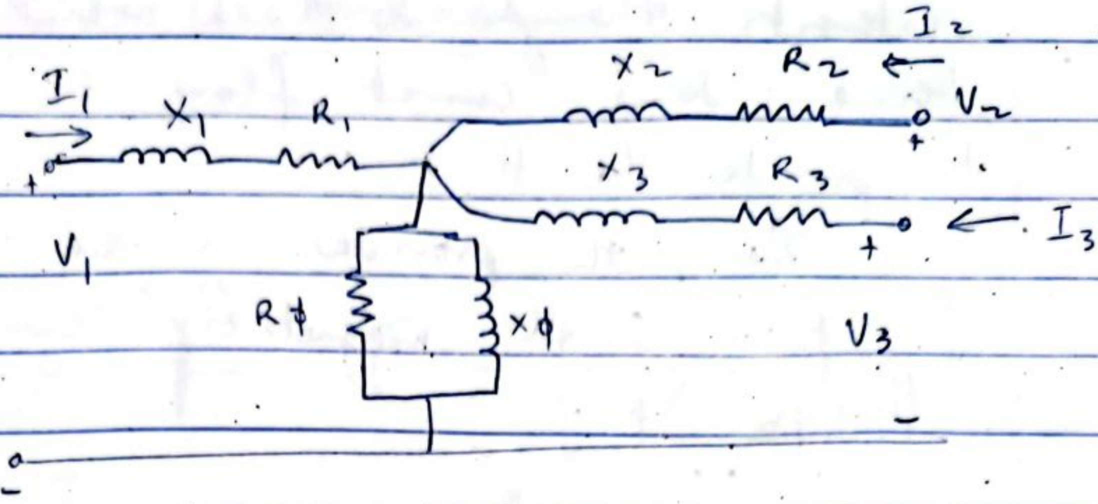
Eqn $*$ & $**$ can be used to construct an equivalent circuit.



(Pu)

Note the point O (junction). The principle can now be extended to n-winding case given this arrangement.

For some studies σ & μ can be assumed to be ∞. However, for other ~~values~~, studies, their values can not be ignored. In such cases, a modified circuit is used



R_1, R_2, R_3 : account for the resistance of the winding conductors.

X_1, X_2, X_3 : account for flux leakage

although 95 – 99% of flux is confined to the core, some escape to surrounding structures and air. This ac leakage flux is proportional to the causing current & also induces a voltage – Faraday. ac voltage and this effect is modeled by inserting a linear inductance as shown

$R\phi, X\phi$: Again, since the core permeability is not infinite, the magnetic field

intensity though small is not zero.
Hence some current flow is necessary to provide the H

The path provided in the circuit for this small magnetizing current is through X_ϕ.

There is internal power loss in the core called core loss due to hysteresis and eddy current phenomena.
This effect is accounted for by the resistance R_ϕ

$R_1\ R_2\ R_3\ X_1\ X_2\ X_3$ are usually small less than 0.05 pu while R_ϕ & X_ϕ is large more than 10 pu.

This information is usually provided by the manufacturer.

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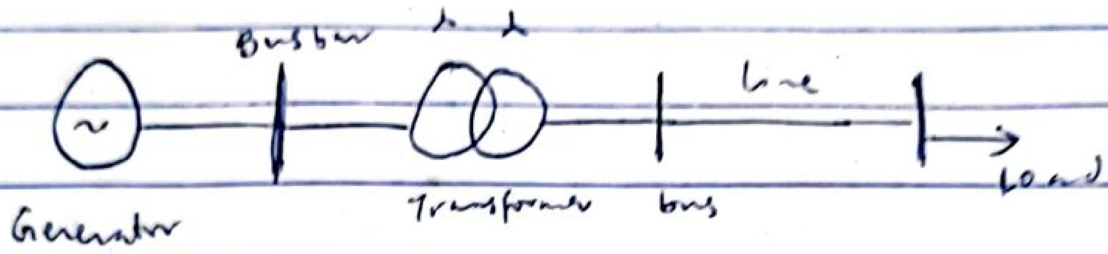
R_1 R_2 R_3 X_1 X_2 X_3 are usually small less than 0.05 pu while R_ϕ & X_ϕ is large more than 10 pu.

This information is usually provided by the manufacturer.

Long Lines > 240km confirm

One line diagram



rotating machine — Gen — motor

Bus (same as node of a circuit diagram)

transformer

Static Load

Δ Delta connection

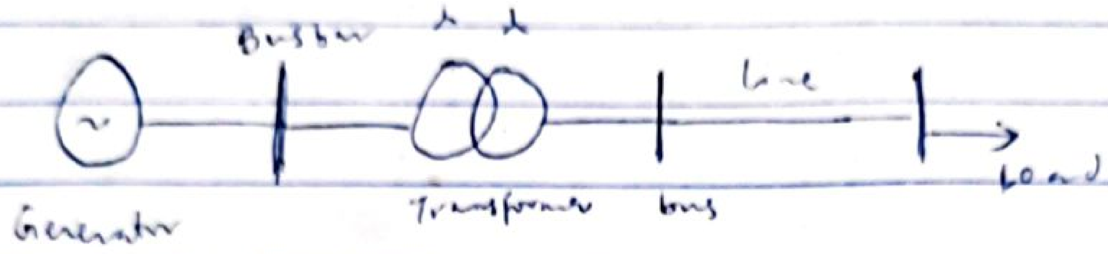
Y Wye connection

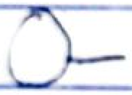
circuit breaker

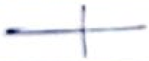
etc

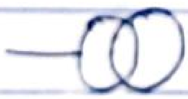
Long Lines > 240Km confirm

One line diagram



 rotating machine 

 Bus (same as node of a circuit diagram)

 transformer

 Static Load

Δ Delta connection

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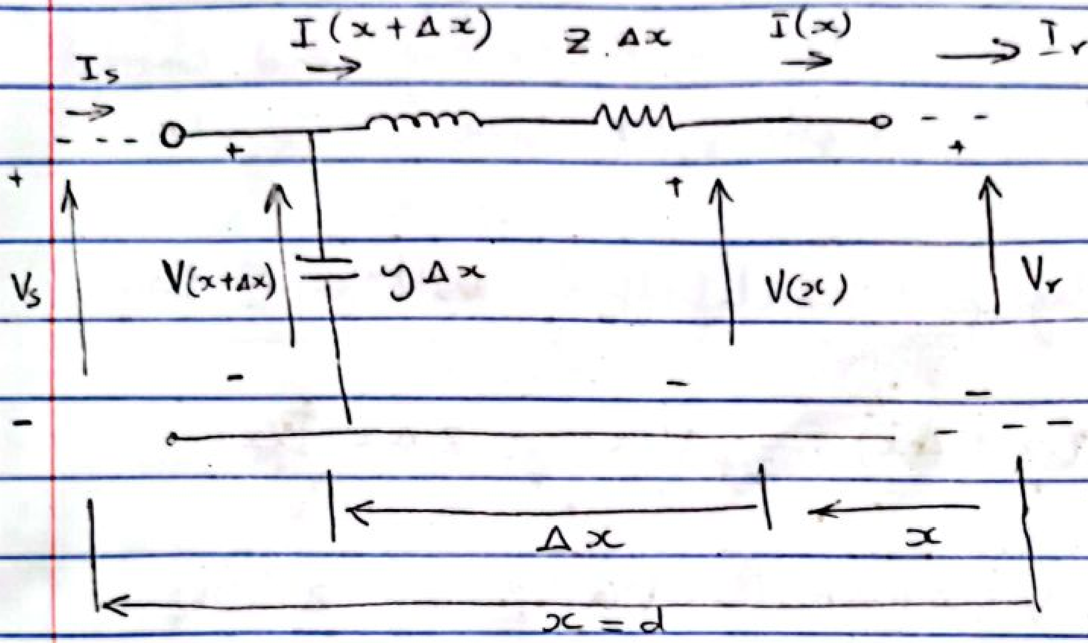
 circuit breaker

etc



x = position along the line, measured positive from the right (receiving end) toward the left (sending end), in metres

$V(x)$ = Phasor voltage at location x on the line

$I(x)$ = Phasor current at location x on the line

z = $R + j\omega L$ = Series impedance per unit length in ohms/m

y = $j\omega C$ = Shunt admittance per unit length in mhos/m or siemens/m

$V_s = V(d)$ = Sending end voltage

$V_r = V(0)$ = Receiving end voltage

$I_s = \bar{I}(d)$ = Sending end Current

$I_r = \bar{I}(0)$ = Receiving end current

d = length of line.

Applying Kirchhoff's voltage Law

$$V(x+\Delta x) = V(x) + z\Delta x \cdot I(x)$$

ie
$$\frac{V(x+\Delta x) - V(x)}{\Delta x} = z\, I(x)$$

limits as Δx tends to zero gives

$$\lim_{\Delta x \to 0} \frac{V(x+\Delta x) - V(x)}{\Delta x} = z\, I(x)$$

$$\frac{d\, V(x)}{dx} = z\, \bar{I}(x) \quad \#$$

Applying Kirchhoff's Current Law

$$I(x+\Delta x) = I(x) + y\Delta x\, V(x+\Delta x)$$

which is the same as

$$\frac{I(x+\Delta x) - I(x)}{\Delta x} = y V(x+\Delta x)$$

limits as Δx tends to zero gives

$$\lim_{\Delta x \to 0} \frac{I(x+\Delta x) - I(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} y V(x+\Delta x)$$

$$\frac{dI(x)}{dx} = y V(x) \qquad \#\#$$

differentiating eqn $\overset{\#}{*}$ & eqn $\overset{\#\#}{**}$ with respect to x to get

$$\left.\begin{aligned} \frac{d^2V(x)}{dx^2} &= z\, dI/dx\,(x) \\ \frac{d^2I(x)}{dx^2} &= y\frac{dV}{dx}(x) \end{aligned}\right\} \quad 1\#$$

To eliminate $dI(x)/dx$ & $dV(x)/dx$ from 1# substitute eqn # & ## to get

$$\frac{d^2V(x)}{dx^2} = zy\,V(x) \qquad 2\#$$

$$\frac{d^2I(x)}{dx^2} = zy\,I(x) \qquad 2\#\#$$

let $\gamma^2 = zy$

$\gamma = \sqrt{zy}$

Since the Units of z & y are ohms/m & Siemens/m, The propagation Constant is in per meter

γ is complex

Let

$\alpha = Re[\gamma]$ = attenuation Constant

$\beta = Im[\gamma]$ = Phase Constant

such that

$$\gamma = \alpha + j\beta$$

replace

Zy by γ^2 in eqn 2# and 2## to get

$$\frac{d^2V(x)}{dx^2} = \gamma^2 V(x)$$

Using P for d/dx () we get

$$P^2 - \gamma^2 = 0$$

$$\therefore \quad P = \pm\gamma$$

and $V(x) = V^+e^{\gamma x} + V^-e^{-\gamma x}$ 2###

The complex constants V^+ & V^- are as a result of solving the second order differential eqn.

If you substitute the result into eqn #

$$Z\,I(x) = \frac{d}{dx}\left[V^+e^{\gamma x} + V^-e^{-\gamma x}\right]$$

$$= \gamma V^+e^{\gamma x} - \gamma V^-e^{-\gamma x}$$

CamScanner

or

$$\frac{z}{\gamma} I(x) = V^{+} e^{\gamma x} - V^{-} e^{-\gamma x} \quad 3\#$$

$Z_c \equiv$ Characteristic impedance $= \sqrt{z/y}$

unit of Z_c - ohms

Z_c is complex

Where series resistance is zero, (Lossless line), $z = j\omega L$ & $y = j\omega C$ the $j\omega$'s cancel out in the expression eqn. for Z_c which gives $Z_c = \sqrt{L/C}$ which is a real number. This is only for lossless lines.

Here we deal with sinusoidal steady state performance & we do not have to neglect the resistance.

$$z/\gamma = z/\sqrt{zy}$$

$$= \sqrt{z/y}$$

$$= Z_c$$

or

$$\frac{Z}{\gamma} I(x) = V^+ e^{\gamma x} - V^- e^{-\gamma x} \quad 3\#$$

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Here we deal with sinusoidal steady state performance & we do not have to neglect the resistance.

$$Z/\gamma = Z/\sqrt{Zy}$$

$$= \sqrt{Z/y}$$

$$= Z_c$$

eqn 3# becomes

$$Z_c I(x) = V^+ e^{+\gamma x} - V^- e^{-\gamma x} \quad 4\#$$

Next we would like to replace V^+ & V^- by V_r & I_r. To do this we apply eqns 2### & 4# at the receiving end $(x=0)$

This result in

$$V(0) = V_r = V^+ + V^- \quad @$$
$$Z_c I(0) = Z_c I_r = V^+ - V^- \quad @@$$

Add eqns @ & @@ & divide by two to get

$$V^+ = (V_r + Z_c I_r)/2 \quad 2@$$

Subtract eqn. @@ from eqn @ and divide by two to get

$$V^- = (V_r - Z_c I_r)/2 \quad 3@$$

eliminating V^+ & V^- from eqn 2### & 4# using expressions for V^+ & V^- in eqn 2@ and 3@

we get

$$V(x) = \frac{V_r + I_r Z_c}{2} e^{\gamma x} + \frac{V_r + I_r Z_c}{2} e^{-\gamma x}$$

$$Z_c I(x) = \frac{V_r + I_r Z_c}{2} e^{\gamma x} - \frac{V_r - I_r Z_c}{2} e$$

which gives

$$V(x) = \left[\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right] V_r + \left[\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right] Z$$

$$Z_c I(x) = \left[\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right] V_r + \left[\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right] I$$

This are eqn containing hyperbolic functions

hence

$$V(x) = \cosh \gamma x \, V_r + Z_c \sinh \gamma x \, I_r$$

$$I(x) = \frac{1}{Z_c} \sinh \gamma x \, V_r + \cosh \gamma x \, I_r$$

eliminating V^+ & V^- from eqn 2### & 4# using expressions for V^+ & V^- in eqn 2@ and 3@

we get

$$V(x) = \frac{V_r + I_r Z_c}{2} e^{\gamma x} + \frac{V_r + I_r Z_c}{2} e^{-\gamma x}$$

$$Z_c I(x) = \frac{V_r + I_r Z_c}{2} e^{\gamma x} - \frac{V_r - I_r Z_c}{2} e^{-\gamma x}$$

which gives

$$V(x) = \left[\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right] V_r + \left[\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right] Z_c I_r$$

$$Z_c I(x) = \left[\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right] V_r + \left[\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right] Z_c I_r$$

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$$V(x) = \cosh \gamma x \, V_r + Z_c \sinh \gamma x \, I_r$$
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we get

$$V(x) = \frac{V_r + I_r Z_c}{2} e^{\gamma x} + \frac{V_r + I_r Z_c}{2} e^{-\gamma x}$$

$$Z_c I(x) = \frac{V_r + I_r Z_c}{2} e^{\gamma x} - \frac{V_r - I_r Z_c}{2} e^{-\gamma x}$$

which gives.

$$V(x) = \left[\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right] V_r + \left[\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right] Z_c I_r$$

$$Z_c I(x) = \left[\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right] V_r + \left[\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right] Z_c I_r$$

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hence

$$V(x) = \cosh \gamma x \, V_r + Z_c \sinh \gamma x \, I_r$$

$$I(x) = \frac{1}{Z_c} \sinh \gamma x \, V_r + \cosh \gamma x \, I_r$$

To obtain sending end quantities, we simply set $x = d$ to get

$$V_s = V(x)\big|_{x=d} = \cosh \gamma d \, V_r + Z_c \sinh \gamma d \, I_r$$

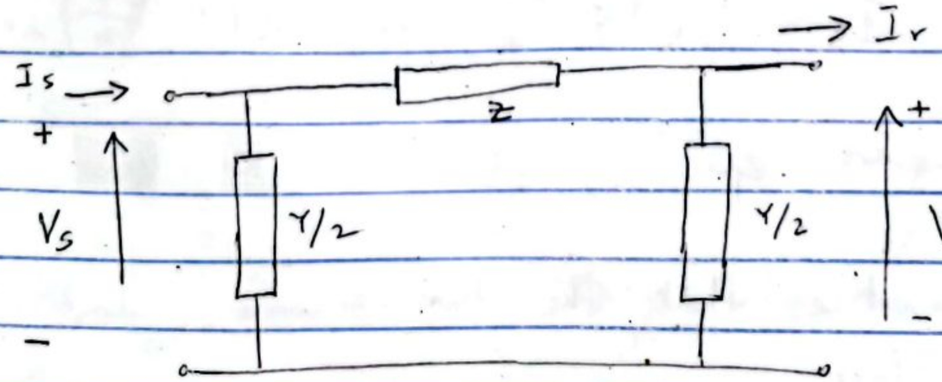
&

$$I_s = I(x)\big|_{x=d} = \frac{1}{Z_c} \sinh \gamma d \, V_r + \cosh \gamma d \, I_r$$

&&

This is adequate to determine line performance at both sending & receiving end terminals. Since we have to connect the line with other power system components, in the network, an equivalent circuit is appropriate; for this purpose.

Consider a π network as shown below



Pi network

Applying Kirchhoff's Voltage law to the circuit, we have

$$V_s = Z\left(I_r + \frac{Y}{2} V_r\right) + V_r$$

ie

$$V_s = \left(1 + \frac{ZY}{2}\right) V_r + Z I_r \quad \boxtimes$$

Applying Kirchhoff's current law, we have

$$I_s = \frac{Y}{2} V_s + I_r + \frac{Y}{2} V_r$$

Substituting eqn $\boxtimes$ in the eqn above we have (ie subs for V_s)

$$I_s = \frac{Y}{2}\left[\left(1 + \frac{ZY}{2}\right) V_r + Z I_r\right] + I_r + \frac{Y}{2} V_r$$

ie

$$I_s = \left(Y + \frac{ZY^2}{4}\right) V_r + \left(1 + \frac{ZY}{2}\right) I_r \quad ▩$$

compare eqn &, && and $\boxtimes$, ▩

We notice that the two become identical if we force

$$z = Z_c \sinh \gamma d \qquad \Delta$$

$$1 + zY/2 = \cosh \gamma d \qquad 2\Delta$$

Substituting Δ in 2Δ to get (& eliminating z)

$$1 + \sinh \gamma d \left[\frac{Z_c Y}{2}\right] = \cosh \gamma d$$

$$Z_c Y/2 = \frac{\cosh \gamma d - 1}{\sinh \gamma d}$$

$$= \frac{e^{\gamma d} + e^{-\gamma d} - 2}{e^{\gamma d} - e^{-\gamma d}}$$

$$Z_c Y/2 = \frac{(e^{\gamma d/2} - e^{-\gamma d/2})^2}{(e^{\gamma d/2} + e^{-\gamma d/2})(e^{\gamma d/2} - e^{-\gamma d/2})}$$

$$= \frac{e^{\gamma d/2} - e^{-\gamma d/2}}{e^{\gamma d/2} + e^{-\gamma d/2}}$$

$$= \tanh \gamma d/2$$

or

$$Y/2 = 1/Z_c \tanh \gamma d/2 = 1/Z_c \tanh \gamma d/2$$

3Δ

• Z_c is a function of the line parameters which is determined once you determine the inductance, capacitance & resistance of the line to get z & y, ie $z(R+jX)$ $Z_c = \sqrt{z/y}$

If $\gamma d << 1$

$$Z = Z_c \sinh \gamma d$$
$$= Z_c (\gamma d)$$

$$Z \approx \sqrt{z/y}\,\sqrt{zy}\; d = \sqrt{\frac{z}{y}}\,\sqrt{zy}\, d \quad ☆$$
$$= z d$$

and

$$Y/2 = 1/Z_c \tanh \gamma d/2$$
$$\approx 1/Z_c (\gamma d/2)$$

$$\approx \sqrt{y/z}\,\sqrt{zy}\; d/2$$
$$\approx y d/2$$

2 ☆

Where and when eqns ☆ & 2 ☆ are used, the resultant circuit model is referred to as the "nominal" pi equivalent circuit.

Notice that in long lines the propagation constant come into play. $\gamma = \sqrt{zy}$. in medium & short lines only the impedance & admittance values are used ie zd & $yd/2$ ie z & y

Generally the approximation is good for transmission lines of length up to 200km. If computers are used for the analysis, this approximation & simplification is not needed. Eqns Δ & 3Δ as well as their associated symmetrical components are used in the analysis.

Line resistance

Temperature

Skin effect

Spiralling

dc resistance of a conductor of uniform material & cross-sectional area

$$R_{dc} = \rho l / A$$

R_{dc} = dc conductor resistance in Ohms

A = Conductor Cross sectional area in m^2

l = Conductor length in m

ρ = Conductor resistivity in meter-ohm

= 2.83×10^{-8} metre-ohm for aluminium at $20°C$

Variation of ρ is $\approx$ approximately linear

$$\rho_2 = \rho_1 \frac{T_2 + T_0}{T_1 + T_0}$$

where T_0 = 228 for aluminium & ρ_1 & ρ_2 are resistivities at temperatures T_1 & T_2, measured in degrees celcius.

Skin Effect:

Tendency of AC current to concentrate at the conductor's surface, thereby increasing the effective resistance. This effect increases with frequency & is observable at 60 Hz

∴ AC resistances are greater than dC resistances

Large power conductors are stranded and strands wound in spiral fashion round conductors center. The strands are somewhat longer than the finished conductor hence a slight increase in resistance.

These effects bring about some percentage increases in resistance. Tables are sometimes provided for practical purposes.

E : electric field strength

Capacitance

Given a line charge density ρ

from Gauss law

$$\int_{surface} \hat{D} \cdot \hat{ds} = q_{enclosed}$$

E is given by

$$E = \frac{\rho}{2\pi \varepsilon x} = \rho / 2\pi \varepsilon x$$

$$\varepsilon = \varepsilon_0 = 1/36\pi \times 10^{-9} \text{ Farad/m}$$

equipotential surfaces are concentric cylinders ~~surrounded~~ surrounding the conductor. The potential difference b/w cylinders, from the position x_a to x_b is

$$V_{ab} = \int_{x_a}^{x_b} E \, dx$$

$$V_{ab} = \int_{x_a}^{x_b} \frac{\rho}{2\pi \varepsilon x} \, dx$$

$$= \rho/_{2\pi\varepsilon} \ln x_b/x_a$$

V_{ab} — Voltage drop from a relative to b ie a is at a higher potential.

The voltage drop b/w two conductors due to a third (k) is given by

$$V_{ij}|_{\rho_k} = V_{ik}|_{\rho_k} + V_{kj}|_{\rho_k}.$$

$$= \frac{\rho_k}{2\pi\varepsilon}\left[\ln\frac{r_k}{D_{ik}} + \ln\frac{D_{jk}}{r_k}\right]$$

$$= \frac{\rho_k}{2\pi\varepsilon} \ln D_{jk}/D_{ik}$$

r_k — radius of Conductor k

D_{ij} — Center to center distance b/w ith & jth conductors.

$$V_{ij} = V_{ij}|_{\rho_1} + V_{ij}|_{\rho_2} + \dots + V_{ij}|_{\rho_i} + V_{ij}|_{\rho_j} + \dots + V_{ij}|_{\rho_n}$$

NB $V_{ij}|_{\rho_1}$ voltage drop b/w i & j as a result of charge ρ in conductor 1

Check case $r_i = r_j$ i.e. r_i by r_j

$$= \frac{1}{2\pi\varepsilon}\Big[\rho_1 \ln \frac{D_{1j}}{D_{1i}} + \rho_2 \ln D_{2j}/D_{2i}$$

$$+ \dots + \rho_i \ln \frac{D_{ij}}{r_i} + \rho_j \ln \frac{r_i}{D_{ji}}$$

$$+ \dots + \rho_n \ln D_{nj}/D_{ni}\Big]$$

i.e

$$V_{ij} = \frac{1}{2\pi\varepsilon} \sum_{k=1}^{n} \rho_k \ln D_{kj}/D_{ki} \quad (D_{kk} = r_k)$$

This is applied only to situations where charge is conserved.

$$\rho_1 + \rho_2 + \dots + \rho_n = 0$$

Consider a four wire three phase conductor

$$V_{in} = \frac{1}{2\pi\varepsilon} \sum_{k=a}^{n} \rho_k \ln \frac{D_{kn}}{D_{ki}}$$

where $i = a, b, c, n$

$k = a, b, c, n$

$D_{ii} = r_i$ = radius of ith conductor for $i = n$, $\ln D_{kn}/D_{kn} = \ln 1 = 0$.

$V_{nn} = 0$ ie $0 = 0$ trivial result.

The remaining remaining 3 eqns have 4 variables eliminating n, gives

$$\rho_n = -(\rho_a + \rho_b + \rho_c)$$

Phase (a)

$$V_{an} = V_a = \frac{1}{2\pi\varepsilon}\left[\rho_a \ln \frac{D_{an}}{r_a} + \rho_b \ln \frac{D_{bn}}{D_{ba}}\right.$$

$$\left. + \rho_c \ln D_{cn}/D_{ca} - (\rho_a + \rho_b + \rho_c) \ln r_n/D_{na}\right)$$

$$V_a = \frac{1}{2\pi\varepsilon}\left[\rho_a \ln D_{an}^2/r_a r_n + \rho_b \ln \frac{D_{bn} D_{na}}{D_{ba} r_n}\right.$$

$$\left. + \rho_c \ln \frac{D_{cn} D_{na}}{D_{ca} r_n}\right]$$

$$V_b = \frac{1}{2\pi\varepsilon}\left(\rho_a \ln D_{an} D_{nb}/D_{ab} r_n + \rho_b \ln D_{bn}^2/r_b r_n\right\}$$

$$+ \rho_c \ln D_{cn} D_{nb}/D_{cb} r_n\Big]$$

$$V_c = \frac{1}{2\pi\varepsilon}\left[\rho_a \ln \frac{D_{an} D_{nc}}{D_{ac} r_n} + \rho_b \ln \frac{D_{bn} D_{nc}}{D_{bc} r_n} + \rho_c \ln D_{cn}^2 / r_c r_n\right]$$

$$\bar{V}_{abc} = [F_{abc}]\, \bar{\rho}_{abc}$$

$[F_{abc}]$ 3 x 3

$$f_{ij} = \frac{1}{2\pi\varepsilon} \ln \frac{D_{in} D_{nj}}{D_{ij} r_n}$$

$i, j = a, b, c$

for sinusoidal steady state analysis both voltages and charge density may be represented by phasors

$$\bar{V}_{abc} = (F_{abc})\, \bar{\rho}_{abc}$$

$$\tilde{\rho}_{abc} = [F_{abc}]^{-1}\, \tilde{V}_{abc}$$

$$= [C_{..}]\, V$$

$$[C_{abc}] = [F_{abc}]^{-1}$$

unit of C_{abc} $\equiv$ Farad/m

Shunt admittance

$$Y = j\omega [C_{abc}]$$

Inductance -

Magnetic effect.

Ampere's Law relates H to i

$$\int_{line} \hat{H} \cdot dl = i_{enclosed}$$

$$H = \frac{i}{2\pi x}$$

H - magnetic field strength or magnetic field intensity

at a distance x external to the conductor

If H is in a constant permeability medium

$$B = \mu H$$

in air $\mu \approx \mu_0 = 4\pi \times 10^{-7}$ H/m

$$\therefore \quad B = \frac{\mu i}{2\pi x}$$

B = flux density

The external flux linking the conductor segment l out to a distance $x = D$ is obtained by integrating from the conductor surface ($x = r$, r = conductor radius) out to the location $x = D$.

$$\lambda' = \int_r^D B\,l\,dx = \frac{\mu i l}{2\pi} \ln D/r$$

{ You are integrating $1/x$ }

A thin walled conductor will have no internal flux linkage since the internal magnetic field is zero. It is possible to compute the radius of an equivalent thin walled conductor r' for any conductor geometry.

It is 'equivalent' in the sense that it has the same total flux linkage as the original conductor

This value r' is referred to as the geometric mean radius (GMR)

Hence, Total flux linkage out to a distance D is (i.e. flux experienced at the distance D from the conductor surface

$$\lambda' = \frac{\mu i l}{2\pi} \ln D/r' \quad \text{Webers}$$

Let flux linkage per unit length be λ

$$\lambda = \lambda'/l \quad \text{webers/m}$$

$$= \mu i/2\pi \ln D/r'$$

given an array of n conductors,

If $i_1 + i_2 + i_3 + i_4 + \cdots + i_n = 0$

for the ith conductor, the flux linkage per unit length, applying superposition. ~~wit~~ will be

$\lambda_i = \lambda_{i1} + \lambda_{i2} + \lambda_{i3} + \ldots + \lambda_{in}$

$\lambda_{ij} \equiv$ flux linkage per unit length around the ith conductor due to current flowing through the jth conductor.

Computing of flux linkage out to some point P outside from the array gives

$$\lambda_{i1} = \frac{\mu_{i1}}{2\pi} \ln D_{1P}/D_{i1}$$

$$\lambda_{i2} = \frac{\mu_{i2}}{2\pi} \ln D_{2P}/D_{i2}$$

$\vdots$

$$\lambda_{ii} = \frac{\mu_{ii}}{2\pi} \ln D_{iP}/r_i'$$

$\vdots$

$$\lambda_{in} = \mu_{in}/2\pi \ln D_{nP}/D_{in}$$

where $D_{ij} = D_{ji}$ = Center to Center b/w conductors i & j $(D_{ii} = r_i')$

$$\lambda_i = \lambda_{i1} + \lambda_{i2} + \lambda_{i3} + \ldots + \lambda_{in}$$

λ_{ij} ≡ flux linkage per unit length around the ith conductor due to current flowing through the jth jth conductor.

Computing flux linkage out to some point P remote from the array gives

$$\lambda_{i1} = \frac{\mu i_1}{2\pi} \ln D_{1P}/D_{i1}$$

$$\lambda_{i2} = \frac{\mu i_2}{2\pi} \ln D_{2P}/D_{i2}$$

$$\vdots$$

$$\lambda_{ii} = \mu i/2\pi \ \ln D_{iP}/r_i'$$

$$\vdots$$

$$\lambda_{in} = \mu i_n/2\pi \ \ln D_{nP}/D_{in}$$

where $D_{ij} = D_{ji}$ = Center to Center b/w conductors i & j $(D_{ii} = r_i')$

$$\lambda_i = \frac{\mu}{2\pi}\left[i_1 \ln \frac{D_{1P}}{D_{i1}} + i_2 \ln \frac{D_{2P}}{D_{i2}} + \dots + i_n \ln \frac{D_{nP}}{D_{in}} \right]$$

$$= \frac{\mu}{2\pi}\left[i_1 \ln \frac{1}{D_{i1}} + i_2 \ln \frac{1}{D_{i2}} + \dots + i_n \ln \frac{1}{D_{in}} + i_1 \ln D_{1p} + i_2 \ln D_{2p} + \dots + i_n \ln D_{np} \right]$$

The total flux linkage per unit length is determined as the point p approaches ∞. As this happens,

$D_{1p} \approx D_{2p} \approx D_{np} \approx D$

$\therefore$

$$\lim_{D \to \infty} (i_1 + i_2 + \dots + i_n) \ln D = 0$$

Since $\Sigma i = 0$

$\therefore$

$$\lambda_i = \frac{\mu}{2\pi}\left[i_1 \ln \frac{1}{D_{i1}} + i_2 \ln \frac{1}{D_{i2}} + \dots + i_n \ln \frac{1}{D_{in}} \right]$$

Similar eqns can be written for all conductors

hence

Matrix form

$$\underbrace{\bar{\lambda}}_{n\times1} = \underbrace{[L]}_{n\times n}\underbrace{\bar{i}}_{n\times1}$$

$L_{ij} = \mu/2\pi \ln 1/D_{ij}$ henry/m

$i,j = 1, 2, 3 \ldots\ldots n$

$D_{ii} = r'_i$

Consider a 3ϕ system, 4 conductors
(3ϕ & a neutral)

$\lambda\,(4\times1)$ $\quad i\,(4\times1)$ $\quad L\,(4\times4)$

(i,j) — index for phases a, b, c & n

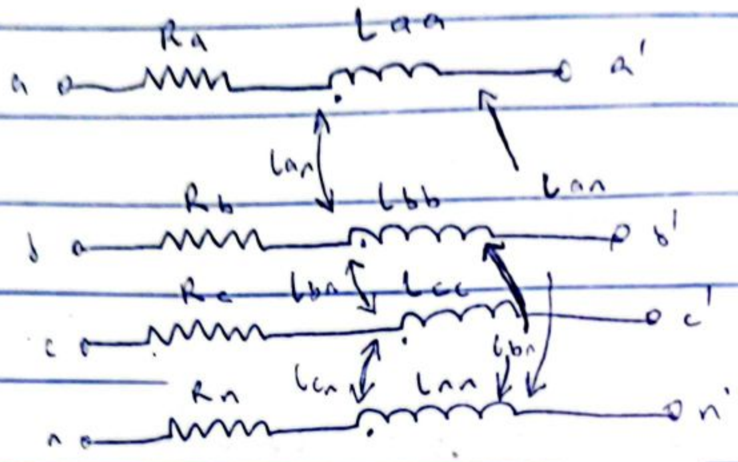
Model for a short line of length L

Let us consider voltages as opposed to flux

$$v = d\lambda/dt$$
$$= L\, di/dt$$

for sinusoidal steady state,

$$V = j\omega L I$$



$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \\ V_{nn'} \end{bmatrix} = l\left([R] + j\omega[L]\right)\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix}$$

$$R = \begin{bmatrix} R_a & 0 & 0 & 0 \\ 0 & R_b & 0 & 0 \\ 0 & 0 & R_c & 0 \\ 0 & 0 & 0 & R_n \end{bmatrix}$$

General equivalent circuit for a four conductor line ~~section~~ section of length L considering serial inductive and resistive effects from the diagram

$$\bar{V}_{ii} = L[Z] I$$

where

Z 4×4

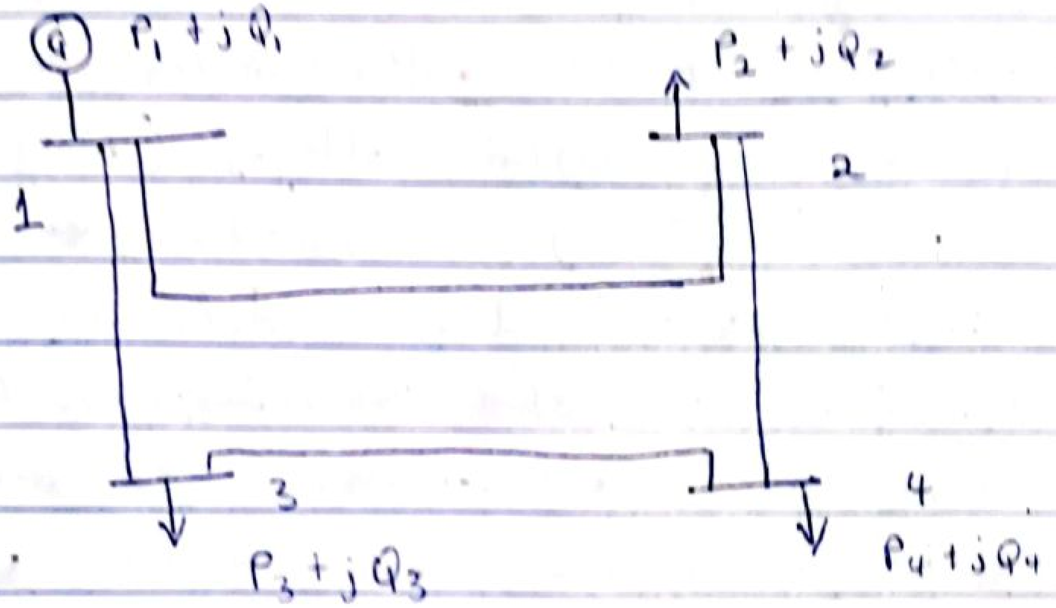
$$Z'_{ii} = R_i + j\omega L_{ii} \quad \text{ohm/m}$$

$$Z_{ij} = j\omega L_{ij} \ (i \neq j) \quad \text{ohm/m}$$

$$L_{ij} = \frac{\mu}{2\pi} \ln 1/D_{ij} \quad \text{henry/m}$$

$$i, j = a, b, c, n$$

LOAD FLOW ANALYSIS



$$V_2 I_2^* = P_2 + jQ_2$$

$$\therefore \quad I_2 = \frac{P_2 - jQ_2}{V_2^*}$$

in terms of self & mutual admittances of the nodes

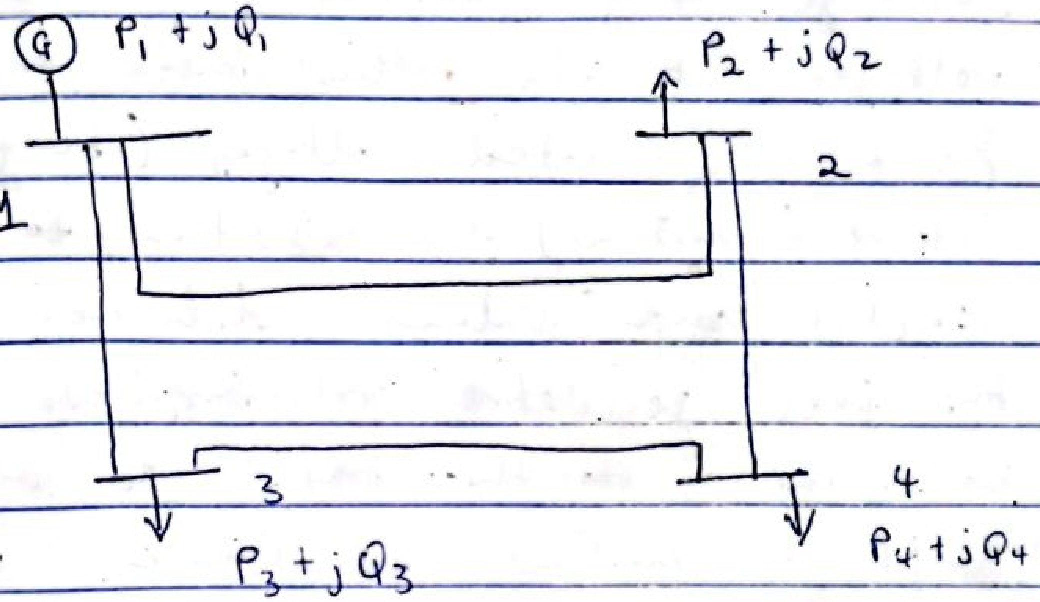
$$\frac{P_2 - jQ_2}{V_2^*} = Y_{21} V_1 + Y_{22} V_2 + Y_{23} V_3 + Y_{24} V_4$$

solving for V_2 gives

$$V_2 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^*} - (Y_{21} V_1 + Y_{23} V_3 + Y_{24} V_4) \right]$$

(A)

LOAD FLOW ANALYSIS



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$$V_2 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^*} - (Y_{21} V_1 + Y_{23} V_3 + Y_{24} V_4) \right]$$

(A)

The idea is to calculate the voltage voltage using estimated values of voltages at the other buses or the previously calculated voltages (being an iterative method) in addition to real & reactive power values delivered to the bus from generators or supplied to the load connected to the bus as well as self & mutual admittances of the nodes.

These quantities are used to form the network equation with the aim of finding the voltage.

Eqn (A) gives a corrected value of V_2 based upon scheduled P_2 and Q_2 when the values estimated originally are substituted for the voltage expressions; on the right side of the eqn.

These improved values of Voltage are

resubstituted in an iterative way until a desired degree of accuracy is attained.

In the GAUSS - SEIDEL METHOD,

As the corrected voltage is found at each bus, it is used in calculating the corrected voltage at the next. This calculation is carried out at each bus except the swing bus throughout the network to complete the first iteration.

This methods involves an iterative solving of linear algebraic eqns.

Note that it is possible for the voltages to converge upon an erroneous solution if the original voltages are widely different from the correct values.

To prevent this original values of reasonable

Magnitude & which do not differ too widely in phase are chosen.

Unwanted solutions can be detected by the experienced engineer easily by inspection of the result since the voltage of the system do not normally have a range in phase wider than 45° and the difference between nearby buses is less than about 10° and often very small.

For N buses the voltage at any bus k when P_k & Q_k are given is

$$V_k = \frac{1}{Y_{kk}}\left(\frac{P_k - jQ_k}{V_k^*} - \sum_{n=1}^{n} Y_{kn} V_n\right)$$

where ~~k~~ $n \neq k$

AA

The value for the voltages on the right side of the equation are the most recently calculated values for the corresponding buses (or the estimated voltage if no iteration has yet been made at that particular bus)

Acceleration factors

These are multipliers designed to improve convergence. The are some constant value that increases the amount of correction to bring the voltage closer to the value it is approaching.

Without this convergence takes a bit longer in a normal Gauss-Seidel method.

The acceleration factor for the real & imaginary values may differ.

A wrong choice would obviously have an opposite effect.

At a bus where voltage magnitude rather than reactive power is specified, a value of reactive power is calculated first to help find the real & imaginary components of the voltage for each iteration.

$$P_k - jQ_k = \left(Y_{kk} V_k + \sum_{n=1}^{N} Y_{kn} V_n \right) V_k^*$$

where $n \neq k$

If we allow $n = k$

$$P_k - jQ_k = V_k^* \sum_{n=1}^{N} Y_{kn} V_n$$

$$Q_k = -I_m \left\{ V_k^* \sum_{n=1}^{N} Y_{kn} V_n \right\} \quad (b)$$

Reactive Power Q_k is evaluated by eqn (b) for the best previous voltage values at the buses, and this value of Q_k is substituted in eqn (AA) to find a new V_k. The components of the new V_k are then multiplied by the ratio of the specified constant magnitude of V_k to the magnitude of the V_k found by eqn (AA). The result is the corrected complex voltage of the specified magnitude.

CS CamScanner

The reason $n \neq k$ is because that expression when $n \neq k$ has been taken out of the Σ sign in one case & expressed separately. In the other case it is used to divide the quantity hence using the Σ sign you have to remove it.

Slack – Reference – Swing bus are the same

Voltage Controlled Bus

Usually has a ~~volt~~ generator connected.

P_{Gi} & V_i can be specified

P_{Gi} is constrained b/w limits

$$P_{Gi\,min} \leq P_{Gi} \leq P_{Gi\,max}$$

Q_{Gi} is also set within limits

$$Q_{Gimin} \le Q_{Gi} \le Q_{Gimax}$$

if Q_{Gi} falls outside the limit, the limit is taken & V_i becomes the unknown.

The calculation is carried out with this understanding & if Q_{Gi} falls b/w. the Limits, V_i is fixed again and the Process continues

Reference:

Generator or tie bus with no constraints

Load bus

all P & Q are known.

Newton Raphson Iterative Method.

The power system is expressed as a function of δ & V. It is then expressed as a taylor series.

It is noted that there are differential quantities in the Taylor series expression. Second order values of the differential & higher order values are ignored, because their effect is minimal.

Incremental change ΔV & $\Delta \delta$ are approximated as error quantities or these differentials.

An iterative method is set in motion such that the difference b/w the calculated ~~voltage~~ complex power & the specified power gives this error quantity. This error is used to improve the assumed values of the voltages & angles. The exercise is ~~contd~~ continued until a given level of accuracy is established.

Consider a guessed solution $x^{(0)}$

Let

$$f(x^{(0)} + \Delta x^{(0)}) = 0$$

if we expand $f(x)$ about x^{0} we obtain

$$f(x^{(0)}) + \Delta x^{(0)} \left(\frac{df}{dx}\right)^{(0)} + \frac{1}{2} (\Delta x^{(0)})^2 \left(\frac{d^2 f}{dx^2}\right)^{(0)} + \cdots = 0$$

where $\Delta x^{(0)}$ is the error associated with the guess $x^{(0)}$

If the error is small, the higher-order terms can be neglected to give

$$f(x^{(0)}) + \Delta x^{(0)} \left(\frac{df}{dx}\right)^{(0)} \approx 0$$

The error can be calculated as

$$\Delta x^{(0)} = -\frac{f(x^{(0)})}{(df/dx)^{(0)}}$$

if we add this error to the original guess, we then have an improved guess, such that

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = x^{0} - \frac{f(x^{(0)})}{(df/dx)^{(0)}}$$

This procedure is repeated until the correct or specified degree of accuracy is attained.

$$x^{(a+1)} = x^{(a)} - \frac{f(x^{a})}{(df/dx)^{(a)}}$$

CamScanner

Consider n- Dimensional Equations

$$x^{(0)} = \begin{bmatrix} x_1^{(0)} \\ \vdots \\ x_n^{(0)} \end{bmatrix}$$

Here only the first derivative terms are considered.

$$f_1(x^{(0)}) + (\partial f_1/\partial x_1)^{(0)} \Delta x_1 + \cdots + \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \Delta x_n \simeq 0$$

$$\vdots$$

$$f_n(x^{(0)}) + (\partial f_n/\partial x_1)^{(0)} \Delta x_1 + \cdots + \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \Delta x_n \simeq 0$$

$$\begin{bmatrix} f_1(x^{(0)}) \\ \vdots \\ f_n(x^{(0)}) \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \\ \vdots & & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \vdots \\ \Delta x_n \end{bmatrix} \simeq \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

Consider n- Dimensional Equations

$$x^{(0)} = \begin{bmatrix} x_1^{(0)} \\ \vdots \\ x_n^{(0)} \end{bmatrix}$$

Here only the first derivative terms are considered.

$$f_1(x^{(0)}) + (\partial f_1/\partial x_1)^{(0)} \Delta x_1 + \dots + \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \Delta x_n \approx 0$$

$$\vdots$$

$$f_n(x^{(0)}) + (\partial f_n/\partial x_1)^{(0)} \Delta x_1 + \dots + \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \Delta x_n \approx 0$$

$$\begin{bmatrix} f_1(x^{(0)}) \\ \vdots \\ f_n(x^{(0)}) \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \\ \vdots & & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \vdots \\ \Delta x_n \end{bmatrix} \approx \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

Using compact matrix - vector notation

$$f(x^{(0)}) + J^{(0)} \Delta x^{(0)} \approx 0$$

hence

$$\Delta x^{(0)} \approx - [J^{(0)}]^{-1} f(x^{(0)})$$

This error vector is added to the original guess and the process repeated in an iterative way

such that

$$x^{(a+1)} = x^{(a)} - [J^{(a)}]^{-1} f(x^{(a)})$$

where

J = is an $n \times n$ matrix of partial derivaties $\partial f_i / \partial x_j$ called a Jacobian matrix

In Power System analysis

$$x^{(0)} = \begin{bmatrix} \delta_2^{(0)} \\ \delta_3^{(0)} \\ \vdots \\ |V_{n-1}^{(0)}| \\ |V_n^{(0)}| \end{bmatrix}$$

notice that the reference bus is not included.

This gives a $(2n-2)$- dimensional state vector

By expanding functions for real and imaginary power ~~f_{P_i}~~ f_{ip} & f_{iq} in Taylor series around the initial guess, we have

$$P_i \approx f_{ip}^{(0)} + \frac{\partial f_{ip}}{\partial \delta_2} \cdot \Delta \delta_2^{(0)} + \cdots + \left(\frac{\partial f_{ip}}{\partial |V_n|} \right)^{(0)} \cdot \Delta |V_n^{(0)}|$$

for $i = 2, \ldots, n$

$$Q_i = f_{iq}^{(0)} + \frac{\partial f_{iq}}{\partial \delta_2} \cdot \Delta \delta_2^{(0)} + \cdots + \left(\frac{\partial f_{iq}}{\partial |V_n|} \right)^{(0)} \cdot \Delta |V_n^{(0)}|$$

$f_{ip}^{(0)}$ & $f_{iq}^{(0)}$ represent real & imaginary (reactive) power leaving bus i or associated with bus i if the bus voltages are set at guessed values.

The difference in power also called Power Mismatch at a given bus

is

$$\Delta P_i^{(0)} \cong P_i - f_{ip}^{(0)}$$
$$\Delta Q_i^{(0)} \cong Q_i - f_{iq}^{(0)}$$

$$\begin{bmatrix} \Delta P_2^{(0)} \\ \vdots \\ \Delta Q_n^{(0)} \end{bmatrix} \approx \begin{bmatrix} \frac{\partial f_{2p}}{\partial \delta_2} & \cdots & \left(\frac{\partial f_{2p}}{\partial |V_n|}\right)^{(0)} \\ \vdots & & \vdots \\ \frac{\partial f_{nq}}{\partial \delta_2} & \cdots & \left(\frac{\partial f_{nq}}{\partial |V_n|}\right)^{(0)} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \vdots \\ \Delta |V_n| \end{bmatrix}$$

$$\Delta U^{(0)} \approx J^{(0)} \cdot \Delta x^{(0)}$$

Guess $x^{(0)}$

Compute $\Delta U^{(0)}$ using eq. &

$$P_i = \sum_{k=1}^{n} |y_{ik}||V_i||V_k| \cos(\delta_k - \delta_i + \gamma_{ik}) \approx f_{iP}$$

$$Q_i = \cancel{\sum_{k=1}^{n}}$$

$$Q_i = -\sum_{k=1}^{n} |y_{ik}||V_i||V_k| \sin(\delta_k - \delta_i + \gamma_{ik}) \approx f_{iQ}$$

for $i = 1, 2, \ldots n$

Note that

$$S_i^* = P_i - j Q_i = V_i^* \sum_{k=1}^{n} y_{ik} V_k$$

for $i = 1, 2, \cdots n$

Compute the Jacobian matrix

Solve the voltage error vector

ie

$$\Delta X^{(0)} \approx (J^{(0)})^{-1} \cdot \Delta U^{(0)}$$

Add voltage errors to the initial guesses to obtain an upgraded state vector

The process is repeated

NB

G-S — voltage error used as a convergence measure

NR — Power mismatch, real & reactive used for this purpose.

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2}$$

$$e^{jx} = \cos x + j \sin x$$

$$e^{-jx} = \cos x - j \sin x$$

$$z = r e^{j\theta}$$

$$z = r(\cos\theta + j \sin\theta)$$

$$\frac{\left(e^{\gamma d/2} - e^{-\gamma d/2}\right)^2}{\left(e^{\gamma d/2} + e^{-\gamma d/2}\right)\left(e^{\gamma d/2} - e^{-\gamma d/2}\right)}$$

They will also cansel out the expressions give the ones above

$$\cos^2 x = 1 - \sin^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$
$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

$$2\sinh^2 x = \cosh 2x - 1$$

$$\sin x \cos x = \tfrac{1}{2}\sin 2x$$

$$e^y = \cosh y + \sinh y$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh x + \sinh x = e^x$$
$$\cosh x - \sinh x = e^{-x}$$

$$e^x = \tfrac{1}{2}(e^x + e^{-x}) + \tfrac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$
$$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$e^2 - a^2 = (e+a)(e-a)$$

$$e^{\gamma d} + e^{-\gamma d} - 2e^{-\gamma d} = 0$$

Square of the two products give the value above

difference of two squares.

The network is now modeled and represented by a number of nodes and lines. The nodes are higher level abstractions of the substations and the lines are deduced from the switching positions of the disconnectors and the circuit breakers. All measurements available are checked for gross errors and a measurement has a corresponding place in the network assigned to it. However, there are not measurements for all lines ends. Thus there are several empty spaces in our modeled network where there are no measurements at all. The measurements also represent various quantities such as node voltages, line currents and line flows. The measurements available are raw and they are not preprocessed at all, just brought into the computer by means means of the acquisition equipment and programs. Hence they are not exact. They are also noisy and as such rather unreliable to use. The errors could be :

- Errors introduced by transducers or metering equipment and A/D converter error, truncation errors, etc.

- errors introduced by the telemetering of measurements, i.e transients or telemeter failure.

It is now possible to use mathemati calculations called State estimation to yield a complete set of calculated measurements over all the network by fitting the raw measurements in the best way after a certain criteria to the given network model. The result from the calculations approximately corresponds to the actual values and the influence of the noise is, if the mathematical requirements ~~after~~ are fulfilled, minimized.

The principal task for this function is yield a description of the power syst ie all non-measured values are estimated as well as and thus the database is complete in some sense. The state estimator achieves this by estimating the so-called state variables of the power system from which all interesting system variables can be calculated.

Before we examine the state estimation procedure further the general features of the state estimator are summarized:

- We get a real-time model of the system, mathematically complete and consistent
- We can estimate non-measurable/non-measured system variables
- We can discover erroneous measurements (measurement equipment)

S_2 = Net power at bus 2

$= -.45 - j.2 = -0.45 - j0.2$

Conjugate of $S_2 = S_2^*$

$= -0.45 + j0.2$

$= 0.492443 \angle 156.037511^\circ$

Net Power is Generated Power − Demand Power

$S = S_G - S_D = P + jQ$

$= (P_G - P_D) + j(Q_G - Q_D)$

- We get the fundamental base, necessary for advanced Control of the Power system with other application functions

Definitions & basic assumptions

The state of a system is described by set of ~~state~~ state variables, which at time t_0 contains all ~~fundamental~~ information about the system which has an influence on the future ($t > t_0$) system behaviour. For a given physical system a number of different state variables can be chosen, for example temperatures, positions, speeds, voltages, currents etc. A convenient choice is the selection of a mini minimum set of variables, thus obtaining a minimum, but sufficient, set of state variables. It is important to emphasize that the state variables

- We get the fundamental base, necessary for advanced control of the power system with other application functions

Definitions & basic assumptions

• The state of a system is described by a set of ~~state~~ state variables, which at time t_0 contains all ~~fundamental~~ information about the system which has any influence on the future $(t > t_0)$

• System behaviour. For a given physical system a number of different state variables can be chosen, for example temperatures, positions, speeds, voltages, currents etc. A convenient choice is the selection of a minimum minimum set of variables, thus obtaining a minimum, but sufficient, set of state variables. It is important to emphasize that the state variables are not necessarily directly accessible, measurable or observable.

The system model that is used for power systems is a node-based model, i.e. only the conditions in the nodes are described by the model. In such a model, power-flows, nodal voltages, line-impedances etc. are the interesting variables. The choice of state variables is rather obvious: if the line-impedances are known, all other values are uniquely defined by the set of complex nodal voltages. Thus the state variables are the ~~abs~~ absolute voltage magnitudes of the nodes, and the angles b/w ~~them~~ the nodes.

The state variables are the variables that are directly calculated by the state estimation function. All other variables, such as power-flows, can then then be calculated from the set of state

(SCADA) Supervisory Control and Data Acquisition

Variables.

Fig 8.8 relates the state estimation function to the previously discussed functions. As can be seen there are several steps to perform before any state estimation can be initiated. The measurements can also be more or less pre-checked. The allocation of measurements can ~~also be more or less pre-checked~~. The leave several lines or zones empty of measurements. There may be no ~~no~~ measurements available from a certain part of the power system due to, eg, a failure in a communication link to an RTU. (Remote terminal unit). A necessary condition for the existence of a solution to the state estimation problem is that the system is observable, ie a possibility ~~of~~ exists of calculating all state variables from the available measurements. If some part of the power system is unobservable, the state estimation program must be able to treat this accordingly - include all observable parts of the system in the estimation algorithm, ~~and~~ exclude the unobservable parts or treat each one separately:

- Exclude non-observable part of the system. Use some method to identify what part(s) of the power network which must be excluded and perform state estimation on the rest of the network.

- Include pseudo-measurements to achieve observability. Sometimes it (is) possible to replace real measurements by intelligent "guesses" and go on with the state estimation.

The first method should be prefered, since it is not advisable to mix real measurements with "guessed" measurements. If, however, many measurements are lost from an important part of the network, the second approach is very reasonable.

Raw Data

↓

Network modelling

- Station Configuration
- network Configuration
- impedance Calculation
- Measurement allocation

↓

Model Validation

- substation check
- network check

↓

State Estimator

Actual state of the Process

$$Z\,I(x) = \frac{d}{dx}\left[V^{+}e^{\gamma x} + V^{-}e^{-\gamma x}\right]$$

$$= \gamma V^{+}e^{\gamma x} - \gamma V^{-}e^{-\gamma x}$$

$$\frac{d^2V(x)}{dx^2} = \gamma^2 V^{+}e^{\gamma x} + \gamma^2 V^{-}e^{-\gamma x}$$

$$= \gamma^2\left(V^{+}e^{\gamma x} + V^{-}e^{-\gamma x}\right)$$

To verify that the solution of the second order differential equation

$\therefore$ is as specified and correct!

$$p^2 = \gamma^2$$

$\therefore$ for $V(x) = V^{+}e^{\gamma x} + V^{-}e^{-\gamma x}$

$$\therefore \frac{\partial^2 V(x)}{\partial x^2} = \gamma^2 V(x)$$

ie $\gamma^2\left(V^{+}e^{\gamma x} + V^{-}e^{-\gamma x}\right)$

as shown above

$$ZI(x) = \frac{d}{dx}\left[V^+e^{\gamma x} + V^-e^{-\gamma x}\right]$$

$$= \gamma V^+e^{\gamma x} - \gamma V^-e^{-\gamma x}$$

$$\frac{d^2V(x)}{dx^2} = \gamma^2 V^+ e^{\gamma x} + \gamma^2 V^- e^{-\gamma x}$$

$$= \gamma^2 \left(V^+e^{\gamma x} + V^-e^{-\gamma x}\right)$$

To verify that the solution of the second order differential equation is as specified and correct.

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for $V(x) = V^+e^{\gamma x} + V^-e^{-\gamma x}$

$$\therefore \frac{\partial^2 V(x)}{\partial x^2} = \gamma^2 V(x)$$

ie $\gamma^2\left(V^+e^{\gamma x} + V^-e^{-\gamma x}\right)$

as shown above

Fig 8.8 state estimation and related functions.

Another basic necessity for the state estimation is that there are must be a redundancy in measurements. If we denote the number of measurements by 'm' and the state variables (two times the number of nodes minus one — the reference angle) by n the redundancy must be greater than one.

in practice it is often 1.5 - 2.0

$$\text{Redundancy} = \frac{m}{n} > 1 \qquad 8.1$$

fig 8.8 state estimation and related functions.

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in practice it is often 1.5 - 2.0

$$\text{Redundancy} = \frac{m}{n} > 1 \qquad 8.1$$

Mathematical formulation of state estimation

The mathematical formulation of state estimation in power systems is based on the assumption that the power system is more or less static.

- Assume that we have a system which is characterized by n state variables which completely describe the "System State". The state variables are denoted by X. X is merely a vector which holds all state variables $X = x_i,\ i = 1, \dots, n$

The 'm' measurements are denoted by 'z'. However, each measurement is corrupted by noise. If the noise is denoted by v the expression relating the measurement to the state can be formulated. The relationship is denoted by h:

$$z_1 = h_1(x_1, x_2, \ldots, x_n) + v_1$$

ie

$$z_1 = h_1(x) + v_1$$

$$\vdots$$

$$z_m = h_m(x) + v_m$$

more compactly we can write

$$z = h(x) + v \qquad 8.2$$

If $h(x)$ is linear, eqn 8.2 can be ~~transformed~~ reformulated:

$$z = Hx + v. \qquad 8.3$$

where H is independent of the state variables x_i. H is called the Measurement Matrix

The state estimation procedure can be summarized as in fig 8.9

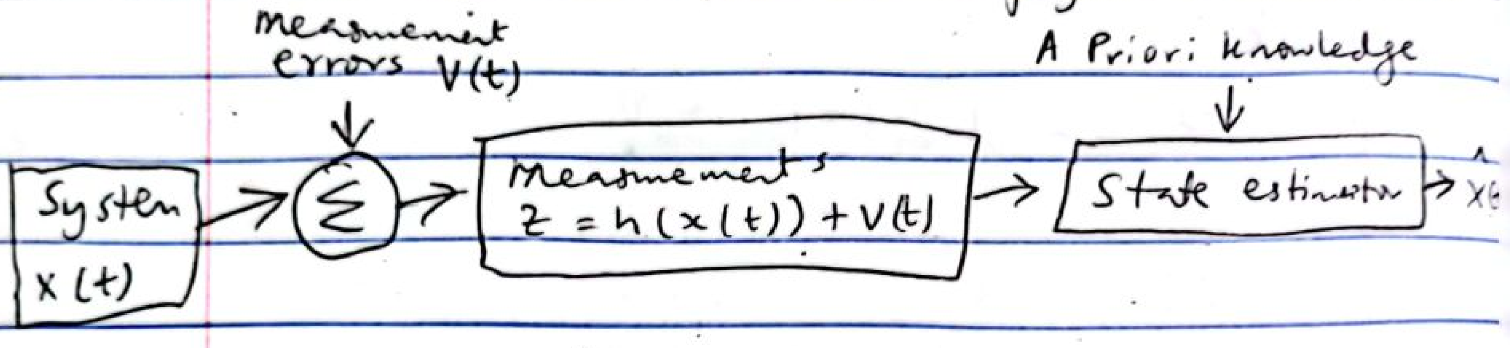

fig 8.9 The state estimation procedure, where $\hat{x}$ is the estimated value of x

Eqn 8.3 relates the measurements z to the state variables via the mathematical expression H if the equations are linear. If the equations are non-linear, as is the case with the power system, the solution requires some iterative method in order to be solved. There are in principle several methods available - maximum likelihood, minimum variance, Kalman approach, least square method etc.

it would perhaps be advantageous to describe all methods available and relate them to each other. This would, however, take the attention from the most used method - the weighted least square method (WLS)

The derivation of the WLS estimator is frequently performed in the literature. The WLS method is generally a scheme where the ~~devo~~ deviations between the measurements and the corresponding equations for the measurements (the estimates) are minimized.

$$J(\hat{x}) = \min_{x} \sum_{i=1}^{m} k_i \left(z_i - h_i(x) \right)^2$$

where $\hat{x}$ is the estimated state variables and k_i constants. These constants form the matrix W. Due to the ~~non-linearity~~ non-linearities the

minimization is an iterative procedure

$$\hat{x}_{n+1} = \hat{x}_n + (H_n^T W H_n)^{-1} H_n^T W [z - h(\hat{x}_n))$$

This means that the state variables are successively ~~approx~~ approximated closer and closer to some value and a convergence criterion determines when the iteration is stopped

The matrix W is called the weighting matrix. It relates the measurements individually to each other. The total sum of $J(x)$ is then influenced by the choice of the elements W. If one measurement is bad it will have less impact on the result compared with a good measurement. This implies that a proper ~~check~~ choice of W is to relate the elements to the quality of the measurements. The selection of identical weights for each of the m measurements, $W = I$, is

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Detection of bad data

The estimator can correct small errors but big ones must be removed.

The difference b/w a measurement and the corresponding estimated value is called the residual

$$r_i = z_i - h_i(\hat{x}) \; ; \; i = 1, \dots, m$$

r_i is the residual for measurement i

The residuals can be checked in different ways to detect the presence of bad data, usually by forming some scalar value ~~which~~ with an "alarm threshold". The methods used for this are of different levels of sophistication

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Some are based on statistical assumptions concerning the nature of the small measurement errors.

The whole procedure is risky. It is not easy to decide if the removal of a measurement influences any other measurement. ~~Collection~~ Collected active and reactive power measurements are for example usually descended from the same transducer. Hence, the measurements and their noise may be correlated. A removal of seemingly bad measurements according to the strategy presented here must therefore be implemented with utmost care.

Implementation:

Convergence Criterion
Successive $\hat{x}_n$ close to each other.

(Power System control Technology) (Torsten Cegrell)

The difference values do not decrease indefinitely, but at a certain ~~stage~~ level rounding errors take over

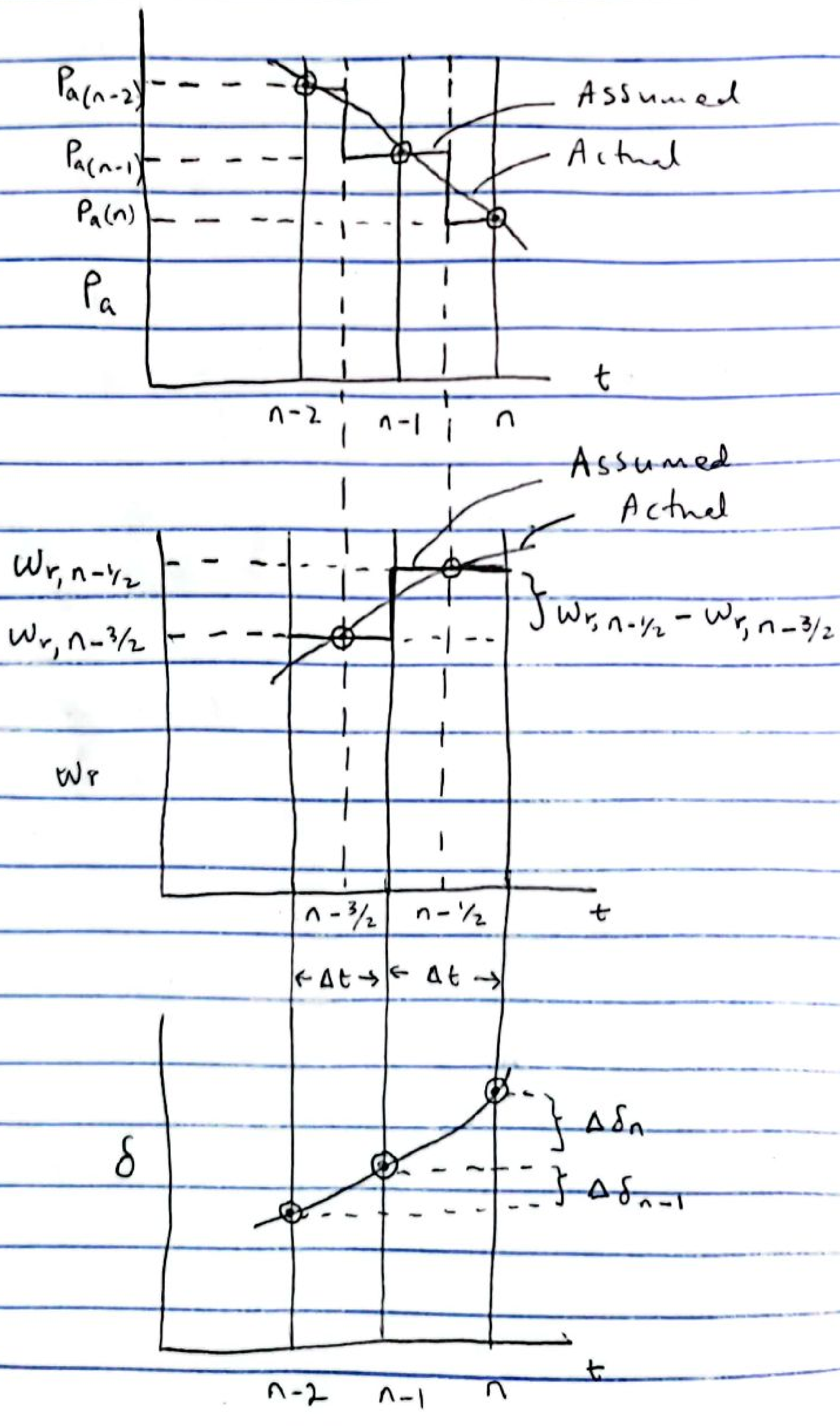
The new formulation of the state estimation function could be to search for for states if such exist, so that the deduced 'measurements' are near the given measurement values; i.e we realise the condition that the state would be the best fit.

We can use the scalar function in eqn 8.4 $J(\hat{x})$ as the conversion variable.

gain matrix

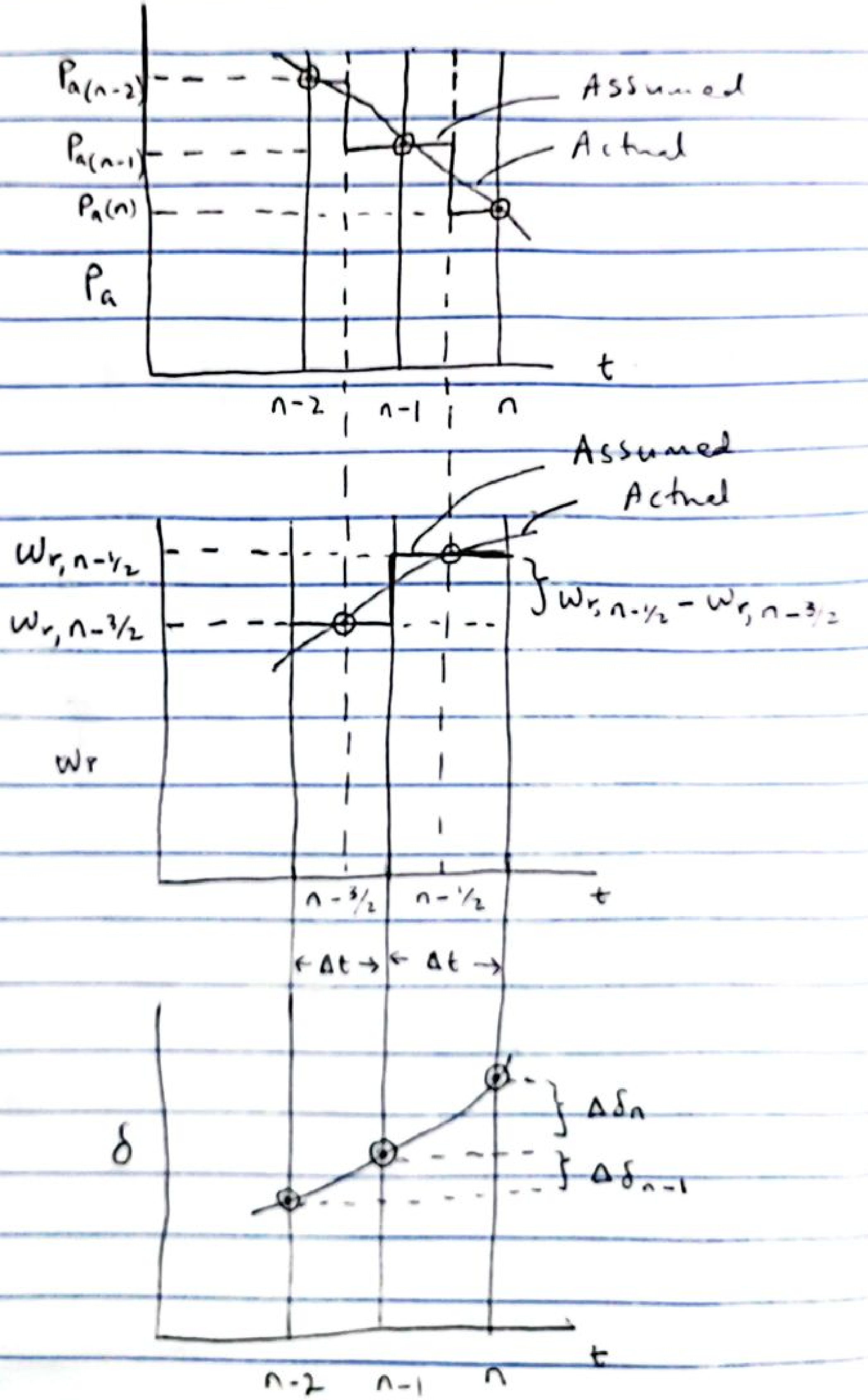
$$(H_n^T W H_n)^{-1} H_n^T W$$

Transient Stability Studies.



Actual & assumed values of P_a, ω_r, and δ as functions of time

Transient stability studies



Actual & assumed values of P_a, ω_r, and δ as functions of time

Step by Step Solution of the Swing Curve.

δ plotted against t. computer

equal area gives you angle.
ie clearing angle

We need clearing time to specify
Circuit breaker settings.

Assumptions.

1 Accelerating power P_a computed at the beginning of an interval is constant from the middle of the preceding interval to the middle of the interval considered. ie

ie because power at beginning & end of interval are different ~~stum~~

hence average is middle of intervals.

Hence the average starts from the middle of one interval & ends at the other middle

2 The ~~agl~~ angular velocity is constant throughout.

any interval at the value computed for the middle of the interval. Of course, neither of the assumptions is true, since δ is changing continuously and both P_a and ω are functions of δ. As the time interval is decreased, the computed swing curve approaches the true curve.

As a result. i.e the smaller the interval the more it approaches per unit value.

Change in speed is given by the product of acceleration and time interval [acceleration x time = speed]

$$\omega_{r,n-1/2} - \omega_{r,n-3/2} = \frac{d^2\delta}{dt^2}\Delta t = \frac{180f}{H} P_{a,n-1} \Delta$$

#

Change in δ over an interval is the product of ω_r for the interval and the time interval.

~~ie Change in ω~~

(angular displacement = angular speed × time)

$$\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \Delta t\, \omega_{r,\, n-3/2}$$

*

during the nth interval

$$\Delta\delta_n = \delta_n - \delta_{n-1} = \Delta t\, \omega_{r,\, n-1/2}$$

* *

eqn ** − eqn * & substituting #
to eliminate ω_r

$$\Delta\delta_n = \Delta\delta_{n-1} + k P_{a,\, n-1}$$

where $k = \frac{180 f}{H} (\Delta t)^2$

Without loss of generality, we will consider the detailed computations for machine 2. Computations to plot the swing curve for machine 1 are left to the student. Accordingly we drop subscript 2 as the indication of the machine number from all symbols in what follows. All our calculations are made in per unit on 100 MVA base. For the time interval $\Delta t = 0.05$ s the parameter k applicable to machine 2 is

$$k = \frac{180 f}{H} (\Delta t)^2 = \frac{180 \times 60}{8.0} \times 25 \times 10^{-4}$$
$$= 3.375 \text{ elect deg}$$

When the fault occurs at $t = 0$ the rotor angle of machine 2 cannot change instantly. Hence from Example 14.9

$$\delta_0 = 16.19°$$

and during the fault,

CamScanner

$$P_e = 0.1545 + 5.5023 \sin(\delta - 0.755^\circ)$$

from Example 14.9

$$P_a = P_m - P_e = 1.6955 - 5.5023 \sin(\delta - 0.755^\circ)$$

At the beginning of the first interval there is a discontinuity in the accelerating power of each machine just before the fault occurs $P_a = 0$ and just after the fault occurs

$$P_a = 1.6955 - 5.5023 \sin(16.19^\circ - 0.755^\circ) = 0.231 \text{ per unit}$$

The average value of P_a at $t = 0$ is $\frac{1}{2} \times 0.2310 = 0.1155$ per unit. We then find

$$kP_a = 3.375 \times 0.1155 = 0.3898^\circ$$

$$P_e = 0.1545 + 5.5023 \sin(\delta - 0.755°)$$

from Example 14.9

$$P_a = P_m - P_e = 1.6955 - 5.5023 \sin(\delta - 0.755°)$$

At the beginning of the first interval there is a discontinuity in the accelerating power of each machine. Just before the fault occurs $P_a = 0$ and just after the fault occurs

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$$kP_a = 3.375 \times 0.1155 = 0.3898°$$

and consequently where we now identify the interval by numerical subscripts

$$\Delta\delta_1 = 0 + 0.3898 = 0.3898^\circ$$

is the change in rotor angle of machine 2 as time advances over the first interval from 0 to Δt. Therefore at the end of the first time interval

$$\delta_1 = \delta_0 + \Delta\delta_1 = 16.19 + 0.3898 = 16.5798^\circ$$

and

$$\delta_1 - \gamma = 16.5798 - 0.755 = 15.8248$$
$$= 15.8248^\circ$$

we then find at $t = \Delta t = 0.05^s$

$$kP_{a,1} = 3.375(P_m - P_c) - P_{max} \sin(\delta_1 - \gamma)$$

$$= 3.375(1.6955 - 5.5023 \sin(15.8248^\circ))$$
$$= 0.6583^\circ$$

and, it follows that the increase in rotor angle over the second time interval is

$$\Delta\delta_2 = \Delta\delta_1 + k P_{a,1} = 0.3898 + 0.6583$$
$$= 1.0481^\circ$$

Hence at the end of the second time interval

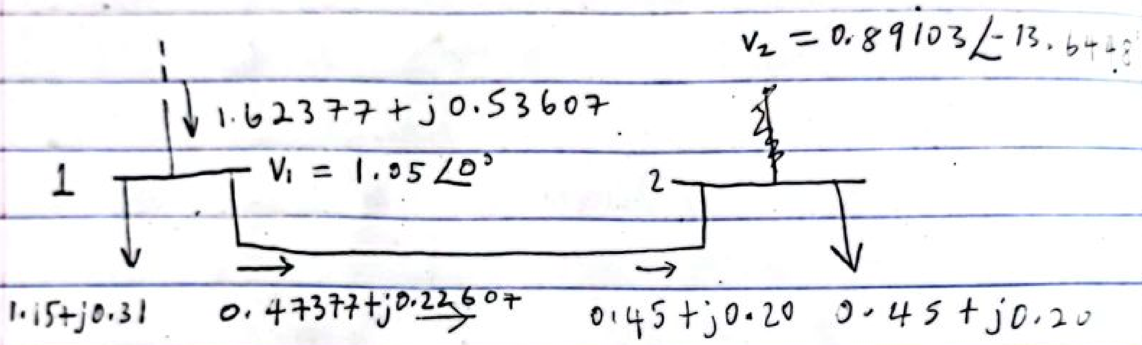
$$\delta_2 = \delta_1 + \Delta\delta_2 = 16.5798 + 1.0481$$
$$= 17.6279^\circ$$

Post fault eqn found in example 14.10 is needed

The term $P_{max} \sin(\delta - \gamma)$, P_a, and δ_n are values computed at the time t shown in the first column but $\Delta\delta_n$ is the change in rotor angle during interval that begins at the time indicated.

for example in the row for $t = 0.10$ s the angle $17.6279°$ is the first value calculated and is found by adding the change in angle during the preceding time-interval (0.05 to 0.10 s) to the angle at $t = 0.05$ s

Next $P_{max} \sin(\delta - \gamma)$ is calculated for $\delta = 17.6279°$. Then, $P_a = (P_m - P_c) - P_{max} \sin(\delta - \gamma)$ and kP_a are calculated. The value of kP_a is $0.3323°$, which is added to the angular change of $1.0481°$ during the preceding interval to find the change of $1.3804°$ during the interval beginning at $t = 0.10$ s. This value added to $17.6279°$ gives the value $\delta = 19.0083$ at $t = 0.15$ s. Note that at 0.25 s the value of $P_m - P_c$ has changed because the fault was cleared at 0.225 s. The angle γ has also changed from 0.755 to $0.847°$



In this case the switched capacitor at bus 2 is out. from the bus loads specified in fig 7.7 we find the bus power.

$$S_1 = P_{G_1} - 1.15 + j(Q_{G_1} - 0.31) \text{ pu}$$

$$S_2 = -0.45 - j0.20 \text{ pu}$$

we need to find

$$V_2 = |V_2| \angle \delta_2$$

$n = 2$

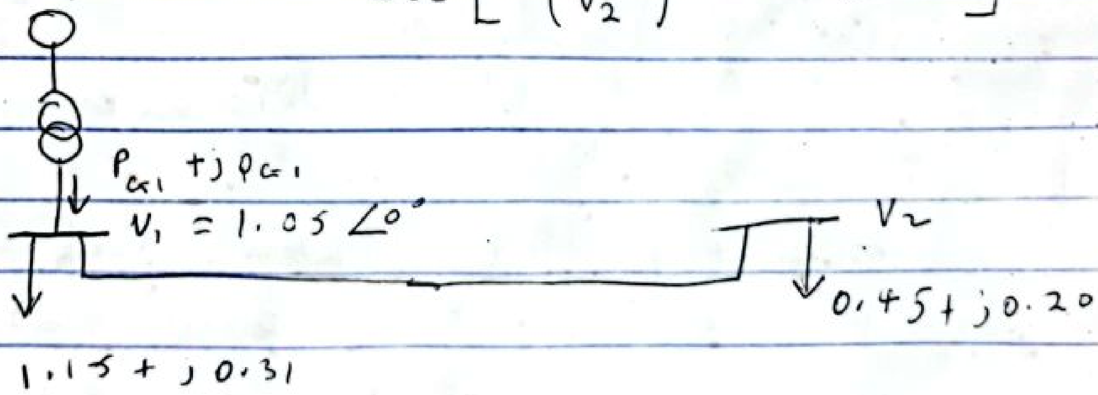
$$V_i^{(a+1)} = \frac{1}{y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(a)})^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} y_{ik} V_k^{(a)}\right]$$

for $i = 2, \ldots, n$

This eqn reduces to

hence we have only to solve one complex eqn. since $n=2$

$$V_2^{(a+1)} = \frac{1}{y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^{(a)})^*} - y_{21} V_1\right]$$



$V_1 = 1.05$ pu

Render the eqn is complex and hence consists of two real eqns.

$$V_2^{(q+1)} = (0.542997 \angle 80.485610)\left(\frac{0.472443\angle 156.037511^\circ}{(V_2^{(q)})^*} - 1.999665\angle 99.1978\right)$$

given a Y_{bus} data

$$Y_{bus} = \begin{bmatrix} 1.841631\angle -80.485610^\circ & 1.904443\angle 99.197819^\circ \\ 1.904443\angle 99.197819^\circ & 1.841631\angle -80.485610^\circ \end{bmatrix} \text{ pu}$$

let us guess that $|V_2|$ will drop by 10% below $|V_1|$

$$\text{ie } |V_2^{(0)}| = 0.95 \text{ pu kV}$$

As 0.45 pu MW (= 135 MW) shall be transmitted via 91 Ω series reactance we use below to ~~fing~~ find the power angle

given kV base 230 kV & MVA base 300 MVA

$$P = \frac{|V_1||V_2|}{X} \sin\delta$$

$$135 = \frac{241 \times 219}{91} \sin\delta$$

$$\therefore \delta = 13.5^\circ$$

hence a reasonable starting value would be

$$V_2^{(0)} = 0.95\angle{-13.5^\circ}\ \text{pu kV}$$

educated guess

hence

$$V_2^{(1)} = (0.542997\angle 80.485610^\circ)\left[\frac{0.492443\angle 156.037511^\circ}{0.95\angle 13.5^\circ} - \cancel{1.999665\angle} - 1.999665\angle 99.197819^\circ\right]$$

$$= 0.902029\angle{-12.682785^\circ}\ \text{pu kV}$$

one additional iteration yields

$$V_2^{(2)} = (0.542997\angle 80.485610^\circ)\ \cancel{\left[\frac{0.492443\angle 156.037511}{0.902029\angle 12.68}\right.}$$

$$\left[\frac{0.492443\angle 156.037511^\circ}{0.902029\angle 12.682785^\circ} - 1.999665\angle 99.197819^\circ\right]$$

hence a reasonable starting value would be

$$V_2^{(0)} = 0.95\angle{-13.5^\circ} \text{ pu kV}$$

Educated guess

hence

$$V_2^{(1)} = (0.542997\angle 80.485610^\circ)\left[\frac{0.492443\angle 156.037511^\circ}{0.95\angle 13.5^\circ} - \cancel{1.999665\angle} - 1.999665\angle 99.197819^\circ\right]$$

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$$V_2^{(2)} = (0.542997\angle 80.485610^\circ)\ \cancel{\left[\frac{0.492443\angle 156.037511}{0.902029\angle 12.68}\right.}$$

$$\left[\frac{0.492443\angle 156.037511^\circ}{0.902029\angle 12.682785^\circ} - 1.999665\angle 99.197819^\circ\right]$$

$$\simeq 0.897226\angle{-13.623066^\circ} \text{ pu kV}$$

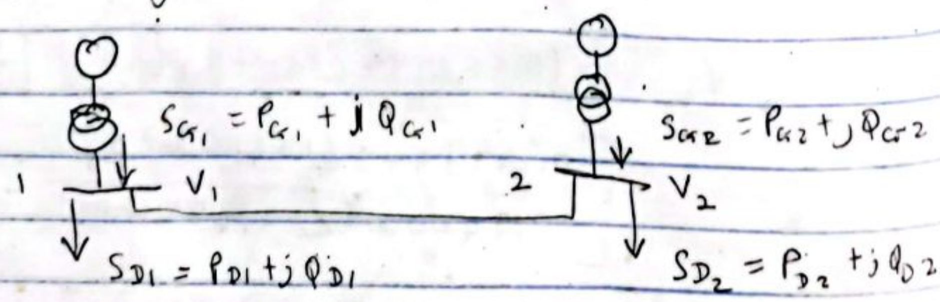
The above computations were made on a hand calculator. If actually programmed on a digital computer voltage convergence to within $\varepsilon = 10^{-4}$ pu will require seven iterations. The final solution will be

$$V_2 = 0.89103 \angle -13.6448^\circ \text{ pu kV}$$

This gives the voltage V_2

- You can use the equations for complex power to get the power flows using the voltage profile as well as the slack bus power.

The transmission line in fig below is a 230kV, 200km line of the phase symmetric type



...ere made on a
...ctually programmed
... voltage convergence
... will require
... final solution will be

$\angle -13.6448°$ pu kV

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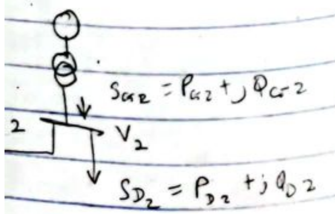

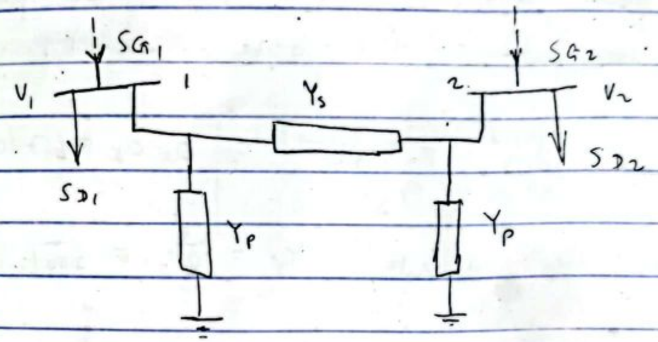

Find the Y_{bus} matrix for the two bus system if we assume the line to be "electrically short". Also express all admittances in per unit referred to the base values:

$$|S_b| = \frac{300}{3} = 100 \text{ MVA/Phase}$$
$$|V_b| = \frac{230}{\sqrt{3}} = 132.79 \text{ kV/phase}$$

We shall carry out all calculations to six significant figures. [Given]

$$R = 0.074 \ \Omega/\text{km}$$
$$\omega L = 0.457 \ \Omega/\text{km}$$
$$\frac{1}{\omega C} = 0.277 \ \text{M}\Omega/\text{km}$$

from the given base values we first compute the base admittance

$$|Y_b| = \frac{300}{230^2} = 0.00567108 \ \mho/\text{phase}$$

We then compute $Y_s = \frac{1}{Z_s} = \frac{1}{200(0.074+j0.457)}$

$$= 0.00172635 - j0.0106614 \ \mho/\text{phase}$$

$$= 0.304413 - j1.879956 \ \text{pu}\ \mho/\text{phase}$$

$y_{11} = y_{22} = y_p + y_s$

$y_{12} = y_{21} = -y_s$

$$Y_p = \frac{1}{-j2770} = j0.00036011 \ \mho/\text{phase}$$

$$= j0.000361011 \ \mho/\text{phase}$$

$$= j0.0636582 \ \text{pu}\ \mho/\text{phase}$$

$$y_{11} = y_{22} = 0.304413 - j1.879956 + j0.0636583$$

$$= 0.304413 - j1.816298$$

$$= 1.841631 \angle -80.485610^\circ$$

$$y_{12} = y_{21} = -0.304413 + j1.879956$$

$$= 1.904443 \angle 99.197819^\circ$$

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$= 0.304413 - j1.816298$

$= 1.841631 \angle -80.485610°$

$y_{12} = y_{21} = -0.304413 + j1.879956$

$= 1.904443 \angle 99.197819°$

Hence

$$Y_{bus} = \begin{bmatrix} 1.841631\angle -80.485610^\circ & 1.90443\angle 99.19 \\ 1.90443\angle 99.197819^\circ & 1.841631\angle -80.4856 \end{bmatrix}$$

Using NR

Use the N-R algorithm to compute the PFE in foregoing example

Remember that bus 2 is a load bus with the bus specifications given below.

$$P_2 = -0.45 \text{ pu MW}$$

$$Q_2 = -0.20 \text{ pu Mvar}$$

The reference bus has the specified voltage $|V_1| = 1.05$ pu

Remember that

$P_2 = P_{G2} - P_{D2}$

π representation $Y_{se}/2 = Y_P$

Hence

$$Y_{bus} = \begin{bmatrix} 1.841631\angle{-80.485610^\circ} & 1.904443\angle{99.197819^\circ} \\ 1.90443\angle{99.197819^\circ} & 1.841631\angle{-80.485610^\circ} \end{bmatrix}$$

using NR

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Remember that

$P_2 = P_{G2} - P_{D2}$

π representation $Y_{e}/2 = Y_p$

Hence

$$Y_{bus} = \begin{bmatrix} 1.841631\angle -80.485610^\circ & 1.904443\angle 99.197819^\circ \\ 1.90443\angle 99.197819^\circ & 1.841631\angle -80.485610^\circ \end{bmatrix}$$

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The reference bus has the specified voltage $|V_1| = 1.05$ pu

Remember that

$P_2 = P_{G2} - P_{D2}$

$$= |y_{21}||V_2||V_1|\cos(\delta_1 - \delta_2 + \gamma_{21}) + |y_{22}||V_2|^2 \cos\gamma_{22} \approx f_{2p}$$

$$Q_2 = Q_{G2} - Q_{D2}.$$

$$= -|y_{21}||V_2||V_1|\sin(\delta_1 - \delta_2 + \gamma_{21}) - |y_{22}||V_2|^2 \sin\gamma_{22} \approx f_{2q}$$

Solve for the two unknowns ie $|V_2|$ and δ_2
The Jacobian matrix is of dimension 2×2 and containing the four partial derivatives

$$J = \begin{bmatrix} \dfrac{\partial f_{2p}}{\partial \delta_2} & \dfrac{\partial f_{2p}}{\partial |V_2|} \\ \dfrac{\partial f_{2q}}{\partial \delta_2} & \dfrac{\partial f_{2q}}{\partial |V_2|} \end{bmatrix}$$

1 is reference hence $\delta_1 = 0$

These derivatives are

$$\frac{\partial f_{2p}}{\partial \delta_2} = -|y_{21}||V_1||V_2|\sin(\delta_2 - \gamma_{21})$$

$$\frac{\partial f_{2p}}{\partial |V_2|} = |y_{21}||V_1|\cos(\delta_2 - \gamma_{21}) + 2|y_{22}||V_2|\cos\gamma_{22}$$

$$\frac{\partial f_{2q}}{\partial \delta_2} = |Y_{21}||V_1||V_2|\cos(\delta_2 - \gamma_{21})$$

$$\frac{\partial f_{2q}}{\partial |V_2|} = |y_{21}||V_1|\sin(\delta_2 - \gamma_{21}) - 2|y_{22}||V_2| \sin\gamma_{22}$$

Here follow the computational steps

Select the same initial guess as before ie $V_2^{(0)} = 0.95\angle -13.5^\circ$

$$X^{(0)} = \begin{vmatrix} -0.235619 \\ 0.950000 \end{vmatrix} \begin{matrix} \text{rad} \\ \text{pu kV} \end{matrix}$$

Bus mismatch

$$\Delta U^{(0)} = \begin{bmatrix} -0.45 + 0.458299 \\ -0.20 + 0.113348 \end{bmatrix} = \begin{bmatrix} 0.008299 \\ -0.086652 \end{bmatrix} \begin{matrix} \text{pu MW} \\ \text{pu Mvar} \end{matrix}$$

Compute the Jacobian

$$J^{(0)} = \begin{bmatrix} 1.752557 & -0.193228 \\ -0.733032 & 1.606169 \end{bmatrix}$$

$$\frac{\partial f_{2q}}{\partial \delta_2} = |Y_{21}||V_1||V_2|\cos(\delta_2 - \gamma_{21})$$

$$\frac{\partial f_{2p}}{\partial |V_2|} = |Y_{21}||V_1|\sin(\delta_2 - \gamma_{21}) - 2|Y_{22}||V_2| \sin\gamma_{22}$$

Here follow the computational steps

Select the same initial guess as before ie $V_2^{(0)} = 0.95\angle{-13.5^\circ}$

$$X^{(0)} = \begin{vmatrix} -0.235619 \\ 0.950000 \end{vmatrix} \begin{matrix} \text{rad} \\ \text{pu kV} \end{matrix}$$

Bus mismatch

$$\Delta U^{(0)} = \begin{bmatrix} -0.45 + 0.458299 \\ -0.20 + 0.113348 \end{bmatrix} = \begin{bmatrix} 0.008299 \\ -0.086652 \end{bmatrix} \begin{matrix} \text{pu MW} \\ \text{pu Mvar} \end{matrix}$$

Compute the Jacobian

$$J^{(0)} = \begin{bmatrix} 1.752557 & -0.193228 \\ -0.733032 & 1.606169 \end{bmatrix}$$

$$(J^{(0)})^{-1} = \begin{bmatrix} 0.600828 & 0.072282 \\ 0.274209 & 0.655588 \end{bmatrix}$$

use the eqn below to find $\Delta x^{(0)}$

$$\Delta x^{(0)} \approx (J^{(0)})^{-1} \cdot \Delta u^{(0)}$$

$$\Delta x^{(0)} = \begin{bmatrix} 0.600828 & 0.072282 \\ 0.274209 & 0.655588 \end{bmatrix} \begin{bmatrix} 0.008299 \\ -0.086652 \end{bmatrix}$$

$$= \begin{bmatrix} -0.001277 \\ -0.054532 \end{bmatrix}$$

update state vector

$$x^{(1)} = \begin{bmatrix} -0.235619 \\ 0.950000 \end{bmatrix} + \begin{bmatrix} -0.001277 \\ -0.054532 \end{bmatrix}$$

$$= \begin{bmatrix} -0.236896 \\ 0.895468 \end{bmatrix} \begin{matrix} \text{rad} \\ \text{pu kv} \end{matrix}$$

The corresponding power mismatch vector equals

$$\Delta U^{(1)} = \begin{bmatrix} -0.001034 \\ -0.005349 \end{bmatrix} \begin{matrix} \text{pu MW} \\ \text{pu MVar} \end{matrix}$$

Repeated computations yield the state vectors

$$X^{(2)} = \begin{bmatrix} -0.238133 \\ 0.891064 \end{bmatrix} \qquad \cancel{X^{(3)} = [-0.23}$$

$$X^{(3)} = \begin{bmatrix} -0.238146 \\ 0.891030 \end{bmatrix}$$

and the power mismatches

$$\Delta U^{(2)} = \begin{bmatrix} -0.000016 \\ -0.000033 \end{bmatrix} \qquad \Delta U^{(3)} = \begin{bmatrix} 0.000000 \\ 0.000000 \end{bmatrix}$$

In three iterations we have converged upon the solution

$$V_2 = 0.89103 \angle -13.1448^\circ \text{ pu kV}$$

Please take note

given $C_n = C_{an} = C_{bn} = \frac{2\pi k}{\ln(D/r)}$ F/m to neutral per phase

$X_c = \frac{1}{2\pi f c}$ [Ω.m] to neutral or per phase

$= \frac{1}{\omega c}$ [Ω.m]

unlike X_L, X_c decreases with increase in distance while the reverse is the case for shunt admittance which is the reciprocal of X_c ie $Y_c = \frac{1}{X_c}$

Remember that ½ the distance is considered when calculating the shunt admittance

Hence given

$X_c = 0.277\,M\Omega/km$

considering 6 km

$Y_p = \frac{1}{X_c} \times \frac{1}{2} = \frac{Y_c}{2}$

Given 200 km

1/2 of 200 km = 100 km

$$Y_p = \frac{100}{-j \cdot 277 \times 10^6} = \frac{1}{-j2770} \ \mho/\text{Phase}$$

$$= j0.000361011 \ \mho/\text{Phase}$$

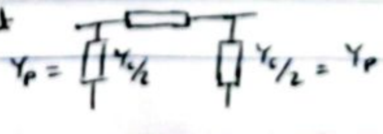
ie $Y_c = \frac{200}{-j \cdot 277 \times 10^6}$

$$Y_p = \frac{Y_c}{2} = Y_c \times \frac{1}{2}$$

$$= \frac{200}{-j \cdot 277 \times 10^6} \times \frac{1}{2} = \frac{1}{-j2770} \ \mho/\text{Phase}$$

$$= j0.000361011 \ \mho/\text{Phase}$$

Remember that



$$P_{T_i} = \sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij})$$

$$Q_{T_i} = \sum_{j=1}^{n} V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij})$$

let $\tilde{x} = \bar{\delta}$

$\tilde{y} = \tilde{V}$

$$\frac{\partial f_i}{\partial x_j} = \frac{\partial P_{T_i}}{\partial \delta_j} = V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial f_i}{\partial x_i} = \frac{\partial P_{T_i}}{\partial \delta_i} = -\sum_{\substack{j=1 \\ j \neq i}}^{n} V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial f_i}{\partial y_j} = \frac{\partial P_{T_i}}{\partial V_j} = V_i Y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial f_i}{\partial y_i} = \cancel{\frac{\partial f_i}{\partial \delta_j}} \frac{\partial P_{T_i}}{\partial V_i} = V_i Y_{ii} \cos(\gamma_{ii}) +$$

$$\sum_{j=1}^{n} V_j Y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial g_i}{\partial x_j} = \frac{\partial Q_{Ti}}{\partial \delta_j} = -V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial g_i}{\partial x_i} = \frac{\partial Q_{Ti}}{\partial \delta_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial g_i}{\partial y_j} = \frac{\partial Q_{Ti}}{\partial V_j} = V_i Y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial g_i}{\partial y_i} = \frac{\partial Q_{Ti}}{\partial V_i} = V_i Y_{ii} \sin(-\gamma_{ii})$$

$$+ \sum_{j=1}^{n} V_j Y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial g_i}{\partial x_j} = \frac{\partial Q_{Ti}}{\partial \delta_j} = -V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial g_i}{\partial x_i} = \frac{\partial Q_{Ti}}{\partial \delta_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial g_i}{\partial y_j} = \frac{\partial Q_{Ti}}{\partial V_j} = V_i Y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial g_i}{\partial y_i} = \frac{\partial Q_{Ti}}{\partial V_i} = V_i Y_{ii} \sin(-\gamma_{ii}$$

$$+ \sum_{j=1}^{n} V_j Y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij})$$

1 a) What is One line Diagram

b) What is P.U. system

Give examples of both

2 Derive from first principles, the circuit model of a synchronous machine. Explain every step

3 Explain the difference b/w a Short line & a long line. What additional factors are taken into consideration when deriving a model for a ~~Short~~ Long line

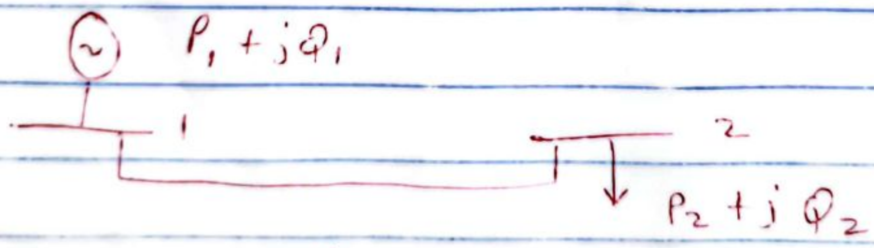


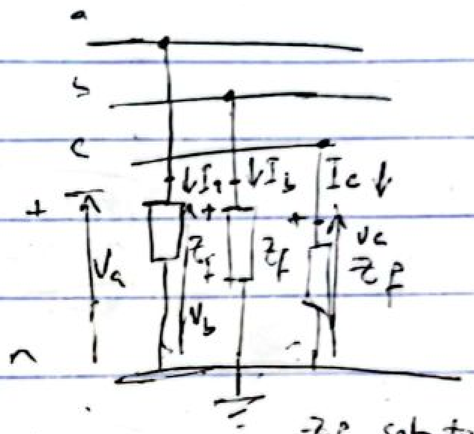
Fig 4

Given Fig 4
derive from first principles the Newton Raphson Iterative method of Solving

the Problem to find the voltage at bus 2 given bus 1 as the reference bus. Assign figures to your expressions and carry out one iteration.

Balanced 3 ϕ fault

$$V_a = I_a Z_f$$



$$V_b = I_b Z_f$$

$$V_c = I_c Z_f$$

Z_f set to zero for fault study.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_f & 0 & 0 \\ 0 & Z_f & 0 \\ 0 & 0 & Z_f \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

[terminated in a fault impedance] though taken to be zero for fault calculations.

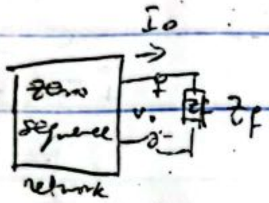
$\cancel{Z_{012} = [T]}$

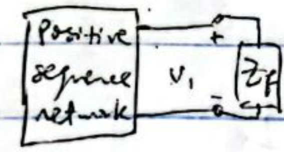
$$Z_{012} = [A]^{-1} \begin{bmatrix} Z_f & 0 & 0 \\ 0 & Z_f & 0 \\ 0 & 0 & Z_f \end{bmatrix} [A]$$

$$= \begin{bmatrix} Z_f & 0 & 0 \\ 0 & Z_f & 0 \\ 0 & 0 & Z_f \end{bmatrix}$$

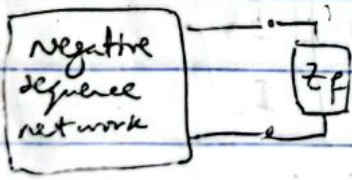
hence

$$V_0 = Z_f I_0$$
$$V_1 = Z_f I_1$$
$$V_2 = Z_f I_2$$





$V_{0,1,2} = Z_{0,1,2} I_{0,1,2}$



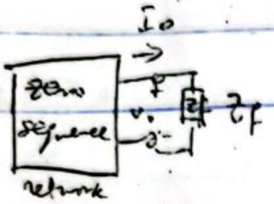
3 ϕ balanced fault sequence network connections.

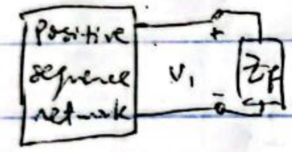
Only the positive sequence is active & hence non trivial. The other two are passive.

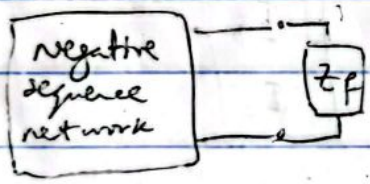
$$V_0 = V_2 = 0$$
$$I_0 = I_2 = 0$$

hence

$$V_0 = Z_f I_0$$
$$V_1 = Z_f I_1$$
$$V_2 = Z_f I_2$$







3 ϕ balanced fault sequence network connections

Only the positive sequence is active & hence non trivial. The other two are passive.

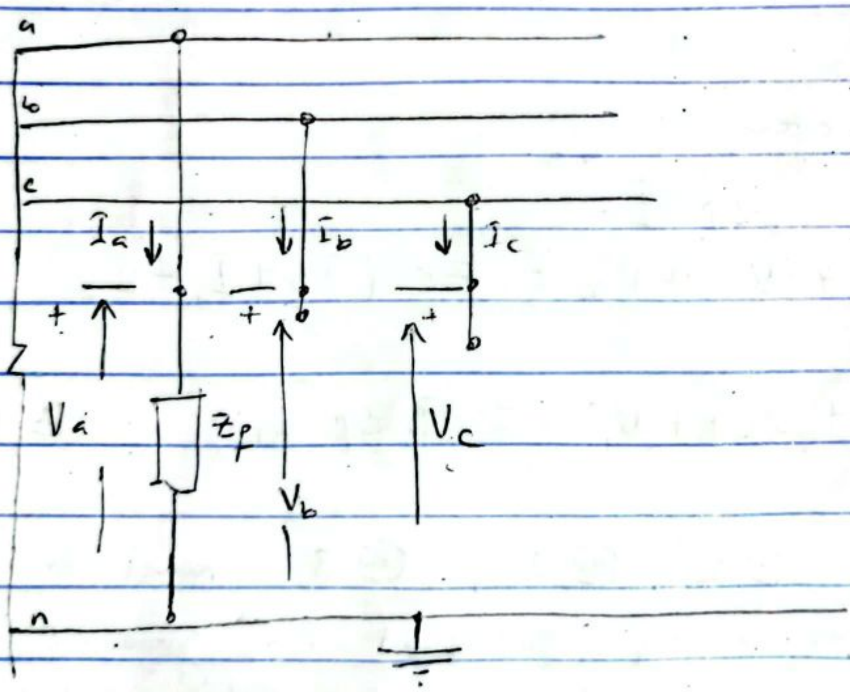
$$V_0 = V_2 = 0$$
$$I_0 = I_2 = 0$$

Single line to Ground fault

$I_b = 0$ terminal conditions

$I_c = 0$

$V_a = I_a Z_f$



$I_b = I_c$

from symmetrical components

ie $I_0 + a^2 I_1 + a I_2 = I_0 + a I_1 + a^2 I_2$

ie

$(a^2 - a) I_1 = (a^2 - a) I_2$

or $I_1 = I_2$ @1

Also

$$\bar{I}_b = \bar{I}_0 + a^2 \bar{I}_1 + a \bar{I}_2 = 0$$

$$\bar{I}_0 + (a^2 + a)\bar{I}_1 = 0$$

$$\bar{I}_0 = -(a^2 + a)\bar{I}_1$$

$$\bar{I}_0 = \bar{I}_1 \qquad @\,2$$

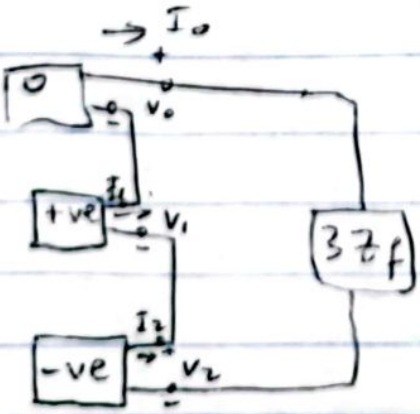
Moreover

$$V_a = Z_f I_a$$

$$V_0 + V_1 + V_2 = Z_f (I_0 + I_1 + I_2)$$

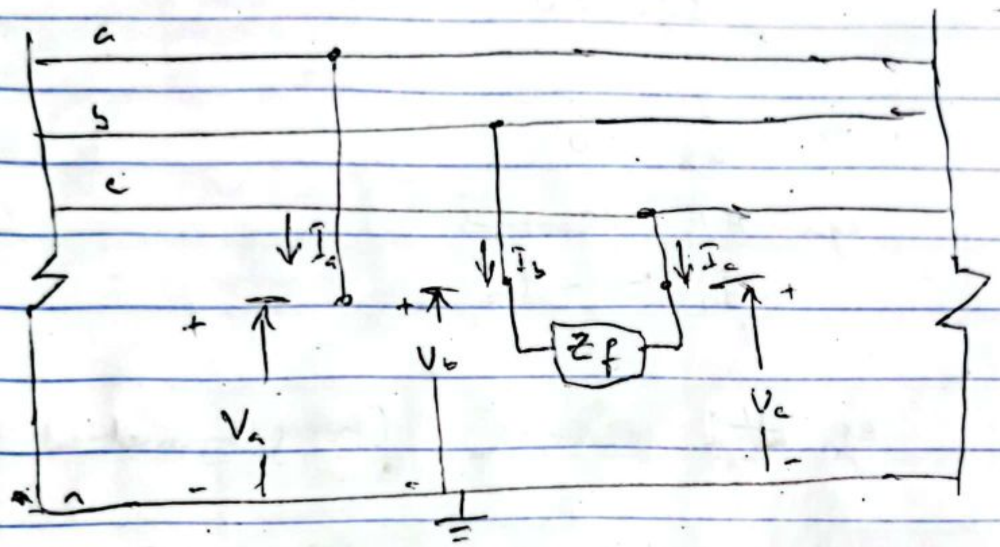
$$V_0 + V_2 + V_1 = 3 Z_f I_1 \qquad @\,3$$

eqns @ 1, @ 2, @ 3 must be simultaneously satisfied in general. This is the case as shown below



Sequence network for the termination of a single line to ground fault

The Line to Line fault



Terminal conditions are such that we can write

$$I_a = 0 \quad \#1$$

$$I_b = -I_c \quad \#2$$

$$V_b = Z_f I_b + V_c \quad \#1$$

from #1

$$I_0 + I_1 + I_2 = 0$$

from #2

$$I_0 + a^2 I_1 + a I_2 = -(I_0 + a I_1 + a^2 I_2) \quad \#3.$$

from #3

$$2I_0 + (a^2 + a)(I_1 + I_2) = 0$$

from #1 $I_a = 0$ apply symmetrical component. ie $I_0 + I_1 + I_2 = 0 \Rightarrow I_0 = -(I_1 + I_2)$
from #3, $(a^2 + a) = -1$
Hence subs for I_0 in #3 gives $3I_0 = 0$

using #1 & #3

$$3I_0 = 0$$

or $$I_0 = 0 \qquad \#2$$

eqn #1 becomes

$$I_1 = -I_2 \qquad \#3$$

eqn #1 becomes (using Symmetrical Components)

$$V_0 + a^2 V_1 + a V_2 = Z_f (I_0 + a^2 I_1 + a I_2) + V_0 + a V_1 + a^2 V_2$$

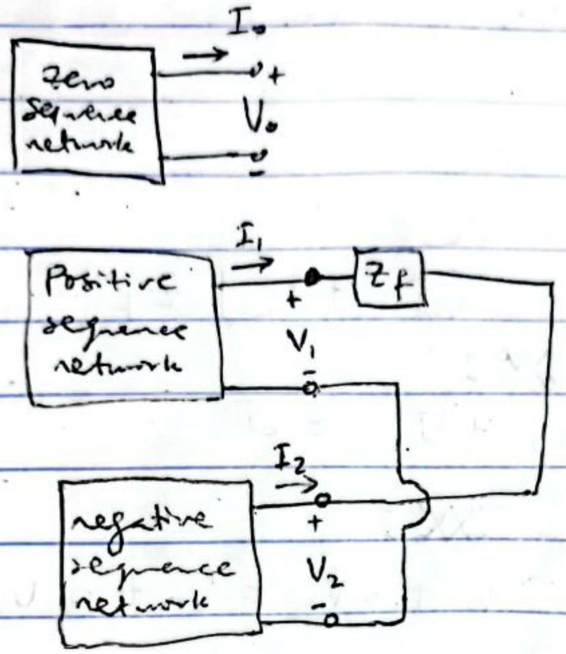
which becomes

$$(a^2 - a) V_1 = (a^2 - a) I_1 Z_f + (a^2 - a) V_2$$

or

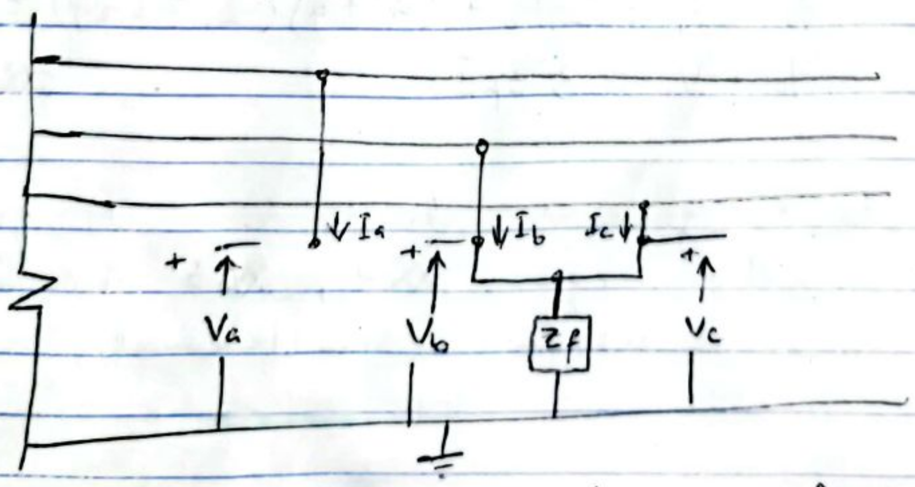
$$V_1 = Z_f I_1 + V_2 \qquad \#4$$

eqns #2, #3 & #4 must be satisfied simultaneously generally.



Sequence network terminations for a line to line fault.

Double line to Ground fault



General double line to ground fault

In this case,

$I_a = 0$ — No. 1

$V_b = V_c$ — No. 2

$V_b = (I_b + I_c) Z_f$ — No. 3

from eqn No. 1

$I_0 + I_1 + I_2 = 0$ — No. 4

from eqn No. 2

$V_0 + a^2 V_1 + a V_2 = V_0 + a V_1 + a^2 V_2$

which gives

$V_1 = V_2$ — No. 5

from eqn No. 3

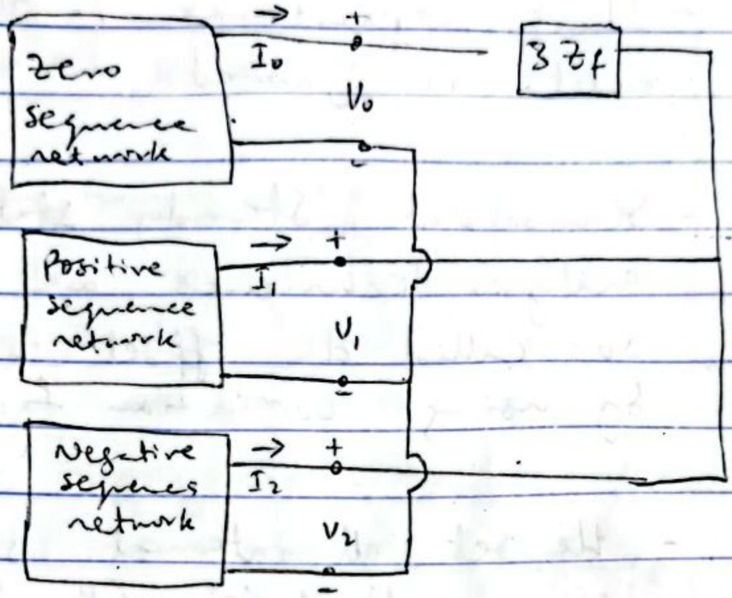
$V_0 + a^2 V_1 + a V_2 = (I_0 + a^2 I_1 + a I_2 + I_0 + a I_1 + a^2 I_2)$

which becomes

$V_0 - V_1 = (2 I_0 + (a^2 + a)(I_1 + I_2)) Z_f$

$V_0 - V_1 = 3 Z_f I_0$ — No. 6

In a general double line to ground fault, eqns No. 4, No. 5 & No. 6 must be satisfied simultaneously.



Sequence network terminations for a double line to ground fault.

Some simplifications in the power system model can make life a lot easier for us during the analysis or while carrying out the study. Some of these include

- We can neglect the shunt elements in the transformer model which account for (magnetization & magnetizing current and core ~~losses~~ . loss

- Shunt Capacitance in the line model is ignored

- Sinusoidal Steady state circuit analysis techniques are used. The so called dc offset is accounted for by using correction factors

- We set all internal system voltage sources to $1\angle 0°$. (for convinience) We select unity by arguing that the system voltage is at its nominal value prior to the ~~phase~~ application of a fault, which is reasonable. The selection of zero phase for one source is ~~of~~ an arbitrary and convenient. Assuming that all sources are in phase and of the same magnitude is equivalent to neglecting prefault load current. When desirable, we shall account for load current using superpossition.

1. First draw the line of the system showing the fault situation.
2. Identify the terminal conditions as a result of the fault condition and write equations describing these.
3. Apply symmetrical component to these terminal condition equations ie write them in terms of their symmetrical components in other words the terminal condition symmetrical component equations
4. Simplify these terminal condition symmetrical component equations until simple circuits can be drawn to illustrate them.
5. Draw the symmetrical component networks and then interconnect the terminals in such a way as to reflect the terminal conditions expressed in symmetrical component terms as shown in the simplified ~~term~~ symmetrical component equations of the terminal conditions.
6. Check the resultant network to ensure that these simplified symmetrical component equations of the terminal conditions are satisfied and that the network is consistent with these eqns.
7. Solve the resultant interconnected symmetrical component.

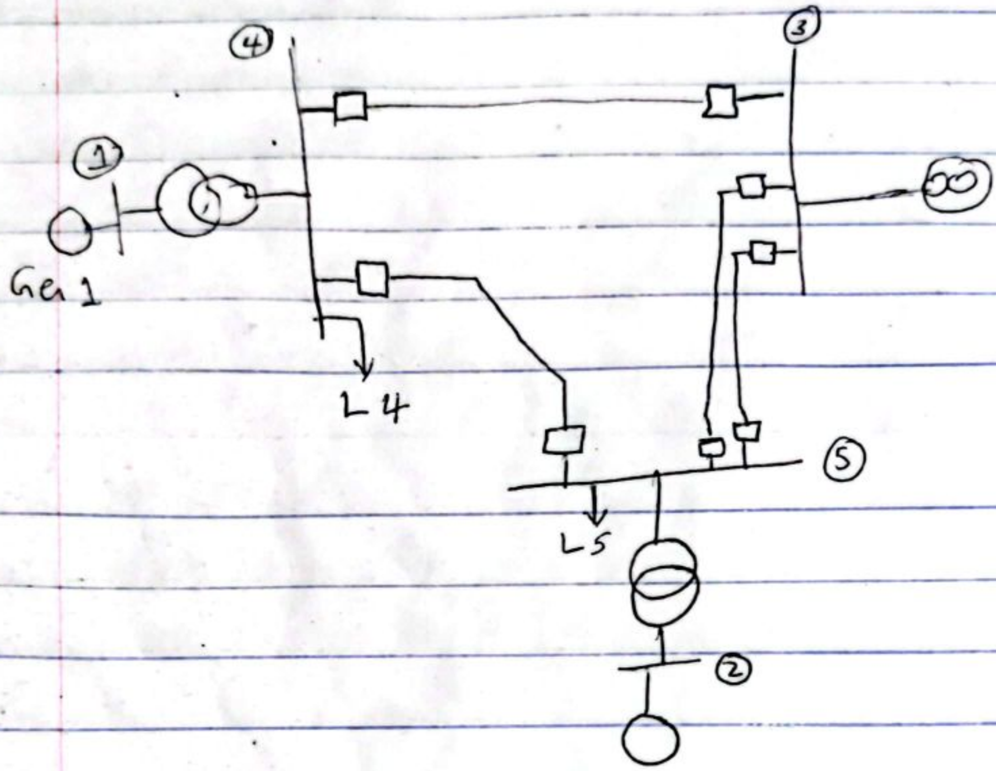
Sequence network, using any of the conventional power system network analysis techniques.

8. The result of the analysis should give a profile of the voltage & current in the resultant network. These would be mainly symmetrical component currents & voltages.

9. Symmetrical Component analysis techniques and relations can then be used to convert these +ve, -ve, & zero sequence values of currents & voltages to phase values as the need arises.

10. Other relevant quantities can also be calculated eg Power etc.

CamScanner



A 60-Hz, 230 kV transmission line has two generators and an infinite bus as shown in the figure above. The transformer and line data are given in Table 14.2 below. A three-phase fault occurs on line 4-5 near bus 4. Using the prefault load-flow solution shown in Table 14.3 determine the swing eqn for each machine during the fault period. The generators

with reactances and H values expressed on a 100 MVA base are described below (as follows:)." The fault is cleared by simultaneously opening the circuit breaker at the ends of the faulted line. Determine the swing eqn for the post-fault period."
In addition, prepare a table showing the steps taken to plot the swing curve for machine 2 for the fault on the 60-Hz system of Example 14.9 and 1

Generator 1 400 MVA, 20kV, $X'_d = 0.067$ per unit
$H = 11.2$ MJ/MVA

Generator 2 250 MVA, 18kV, $X'_d = 0.10$ per unit
$H = 8.0$ MJ/MVA

Table 14.2 Line and transformer data for Example above all values in per unit on 230-kV, 100-MVA base.

Table 14.2 Line and transformer data for Example 14.9, all values in per unit on 230-kV, 100-MVA base

Bus to Bus		Series Z		Shunt Y
		R	X	B
Trans	1 - 4	....	0.022	
Trans	2 - 5	....	0.040	
Line	3 - 4	0.007	0.040	0.082
Line	3 - 5 (1)	0.008	0.047	0.098
Line	3 - 5 (2)	0.008	0.047	0.098
Line	4 - 5	0.018	0.110	0.226

Table 14.3 Bus data and prefault Load-flow Values in per unit on 230kV, 100-MVA base

		Generation		Load	
Bus	voltage	P	Q	P	Q
1	$1.030\angle 8.88^\circ$	3.500	0.712		
2	$1.020\angle 6.38^\circ$	1.850	0.298		
3	$1.000\angle 0^\circ$	....	...		
4	$1.018\angle 4.68^\circ$	....	...	1.00	0.44
5	$1.011\angle 2.27^\circ$	...	...	0.50	0.16

[illegible]
[illegible] Office of Statistics

The art of Electronics
Paul Horowitz
Winfield Hill